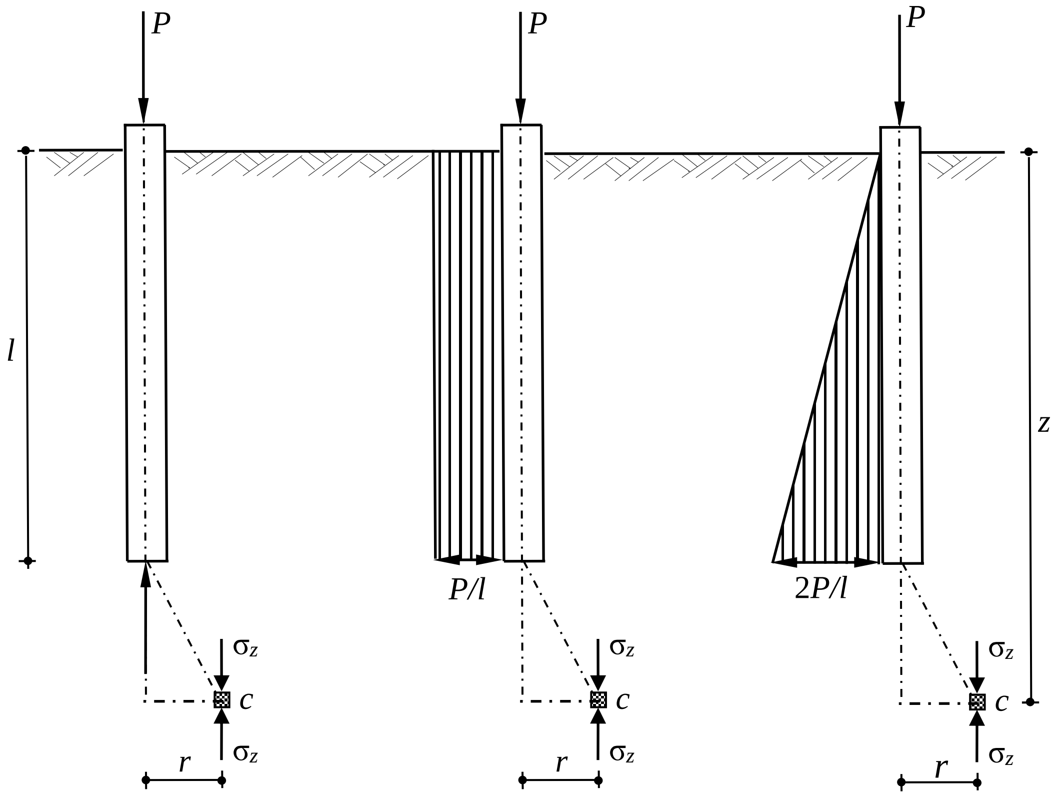
Stress Coefficients According to *GEDDES* by the Program *GEO Tools*



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2022

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Preface

Various problems in Geotechnical Engineering can be investigated by the program *GEO Tools*. The original version of the program *GEO Tools GEOTEC Office* was developed by Prof. M. Kany, Prof. M. El Gendy and Dr. A. El Gendy. After the death of Prof. Kany, Prof. M. El Gendy and Dr. A. El Gendy further developed the program to meet the needs of practice.

This book describes the essential equations used in *GEO Tools* to obtain stresses in soil due to applied load below the ground surface according to *GEDDES* with some verification examples. Besides, the book presents some pile foundation problems solved by hand calculation using *GEDDE'S* solution.

Stress in soil for applied load below the ground surface is obtained by *Mindlin's* solution but not by *Boussinesq's* solution. Unfortunately, *Mindlin's* solution is difficult to apply for hand calculation. In this case, it is convenient to use stress coefficients to determine the vertical stress. For practical application, the complex stress coefficients according to *GEDDES* may be obtained by the program *GEO Tools*. These coefficients can be tabulated for different loading geometries. The tabulated values are easy to use and enable a wide range of loading system to be analyzed. The *GEDDE'S* solution is useful to easily compute manually the elastic settlement or the consolidation settlement for pile foundation problems from stresses.

4 Stress Coefficients According to *GEDDES*

4.1 Introduction

In 1936 Raymond *Mindlin* presented a mathematical solution for determining stresses and displacements in soil due to a point load acting beneath the surface of semi-infinite mass. *Mindlin's* solution are often employed in numerical analysis of piled foundations and may have other applications in Geotechnical Engineering such as study the interaction between foundations and ground anchors or buried structures.

4.2 Vertical stress in soil due to a point load

The vertical stress σ_z [kN/m^2] at point *c* due to a concentrated load *P* [kN] acting at point *j* beneath the surface of a semi-infinite mass, Figure 4.1, according to *Mindlin's* solution can be expressed as:

$$\sigma_z = \frac{P}{8\pi\,(1-\nu_s)}\left(\frac{-(1-2\nu_s)(z-l)}{R_1^3} + \frac{(1-2\nu_s)(z-l)}{R_2^3} - \frac{3(z-l)^3}{R_1^5} - \frac{3(3-4\nu_s)\,z(z+l)^2 - 3(z+l)(5z-l)}{R_2^5} + \frac{30\,z\times l\,(z+l)^3}{R_2^7}\right) \tag{4.1}$$

where:

$R_1 = \sqrt{r^2+(z-l)^2}$, $R_2 = \sqrt{r^2+(z+l)^2}$ and

l Depth of the point load *P* [kN] from the surface, [m].
z Depth of the studied point *i* from the surface, [m].
r Radial distance between points *i* and *j*, [m].
z-l Vertical distance between points *i* and *j*, [m].
z+l Vertical distance between points *i* and *k*, [m].
ν_s *Poisson's* ratio of the soil, [-].

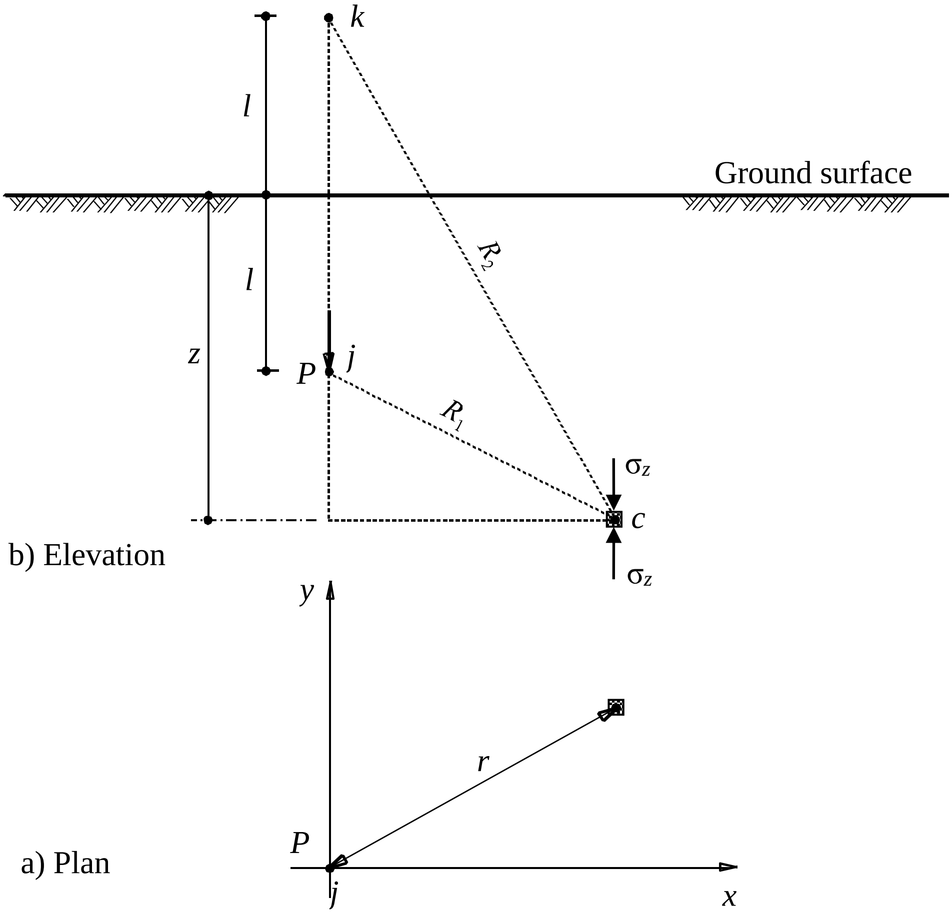

Figure 4.1 Geometry of *Mindlin's* problem

The above *Mindlin's* equation for determining the vertical stress in soil for applied load below the ground surface is difficult to apply for hand calculation. It is convenient to use stress coefficients in this situation to determine the vertical stress. For numerical evaluation and use *Mindlin's* equation is best expressed in dimensionless form. *Geddes* (1966) modified this equation to dimensionless form, which is described in the next paragraph

.

4.3 Vertical stress coefficients according to *Geddes*

Based on *Mindlin's* solution, *Geddes* (1966) presented a solution to obtain the vertical stress at a point in the soil due to applied load below the ground surface using a stress coefficient.

In the solution, three cases of vertical loadings are considered (Figure 4.2):

Case1: Point (end bearing) load. *Mindlin's* case.
Case2: Uniform skin friction.
Case3: Linear variation of skin friction.

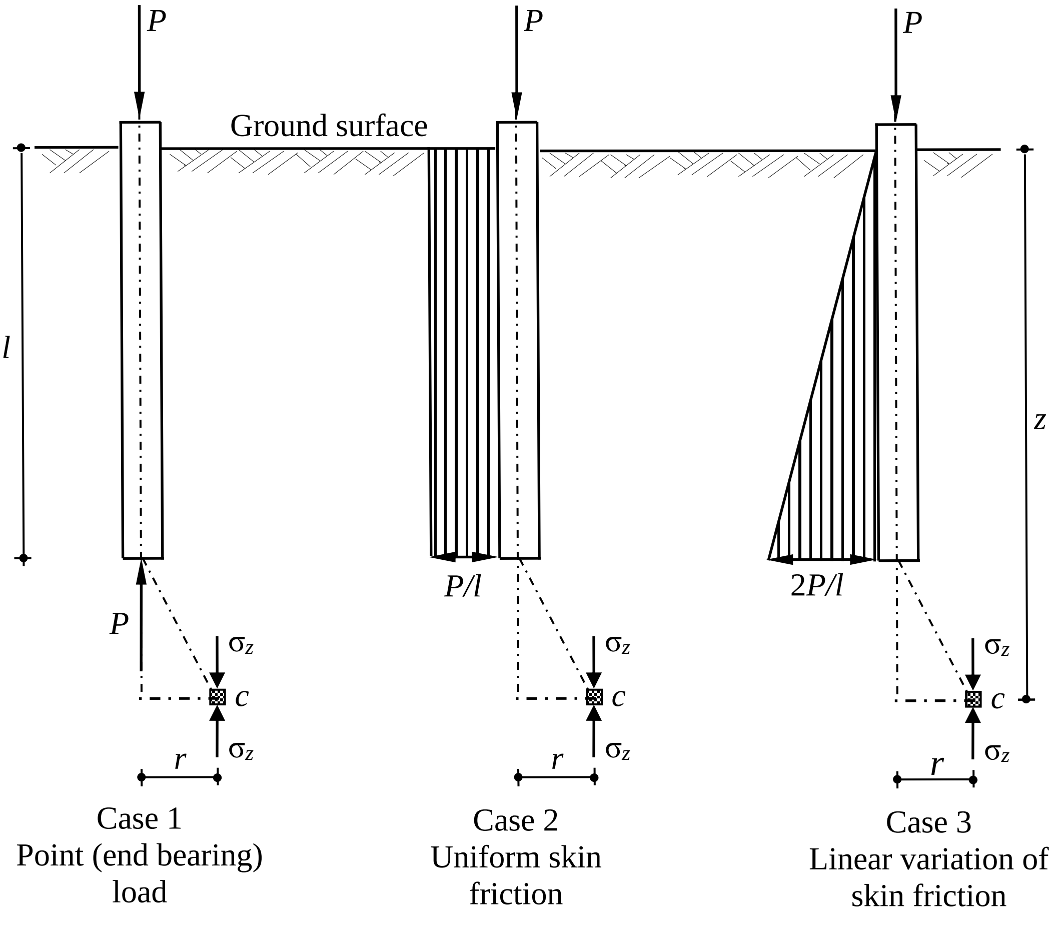

Figure 4.2 Pile-soil system for the evaluation of soil stresses [After *Geddes* (1966)]

According to *Geddes* (1966), the vertical stress due to the applied load is expressed by a very simple equation:

$$\sigma_z = \frac{P}{l^2} K_z \tag{4.2}$$

where:

P	Load carried by the pile, [kN].
l	Pile length, [m]
K_z	Dimensionless influence coefficient, function only of *Poisson's* ratio, [-]

The vertical stress in the soil is expressed in dimensionless form by substituting the following expressions:

$$N = \frac{r}{l},\quad M = \frac{z}{l},\quad A = \sqrt{N^2 + (M-1)^2}\ ,\quad B = \sqrt{N^2 + (M+1)^2}\ ,\quad F = \sqrt{N^2 + M^2}$$

The vertical stress coefficient K_z for case 1 of point load (end-bearing) is given by:

$$K_z = \frac{1}{8\pi\,(1-\nu_s)}\left(\frac{-(1-2\nu_s)(M-1)}{A^3} + \frac{(1-2\nu_s)(M-1)}{B^3} - \frac{3(M-1)^3}{A^5}\right.$$
$$\left. - \frac{3(3-4\nu_s)\,M(M+1)^2 - 3(M+1)(5M-1)}{B^5} + \frac{30\,M\,(M+1)^3}{B^7}\right) \qquad (4.3)$$

The vertical stress coefficient K_z for case 2 of constant skin friction is given by:

$$K_z = \frac{1}{8\pi(1-\nu_s)}\left(\frac{-2(2-\nu_s)}{A} + \frac{2(2-\nu_s) + 2(1-2\nu_s)\frac{M}{N}\left(\frac{M}{N}+\frac{1}{N}\right)}{B}\right.$$
$$- \frac{(1-2\nu_s)^2\left(\frac{M}{N}\right)^2}{F} + \frac{N^2}{A^3} + \frac{\left(4M^2 - 4(1+\nu_s)\left(\frac{M}{N}\right)^2 M^2\right)}{F^3}$$
$$\frac{4M(1+\nu_s)(M+1)\left(\frac{M}{N}+\frac{1}{N}\right)^2 - (4M^2 + N^2)}{B^3}$$
$$\left. + \frac{6M^2\left(\frac{M^4 - N^4}{N^2}\right)}{F^5} + \frac{6M\left[M \times N^2 + \frac{1}{N^2}(M+1)^5\right]}{B^7}\right) \qquad (4.4)$$

The vertical stress coefficient K_z for case 3 of a linear variation of skin friction is given by:

$$K_z = \frac{1}{4\pi(1-\nu_s)}\left(\frac{-2(2-\nu_s)}{A} + \frac{2(2-\nu_s)+(4M+1)-2(1-2\nu_s)\left(\frac{M}{N}\right)^2(M+1)}{B}\right.$$

$$+\frac{2(1-2\nu_s)\left(\frac{M}{N}\right)^2-8(2-\nu_s)M}{F}+\frac{M\times N^2(M-1)^3}{A^3}$$

$$\frac{4\nu_s N^2\times M+4M^3-15N^2\times M-2(5+2\nu_s)\left(\frac{M}{N}\right)^2(M+1)^3+(M+1)^3}{B^3}$$

$$+\frac{2(7-2\nu_s)M\times N^2-6M^3+2(5+2\nu_s)\left(\frac{M}{N}\right)^2 M^3}{F^5}$$

$$+\frac{6M\times N^2(N^2-M^2)+12\left(\frac{M}{N}\right)^2(M+1)^5}{B^5}-\frac{12\left(\frac{M}{N}\right)^2 M^5+6M\times N^2(N^2-M^2)}{F^5} \qquad (4.5)$$

$$\left.+2(2-\nu_s)\ln\left(\frac{A+M-1}{F+M}\quad\frac{B+M+1}{F+M}\right)\right)$$

4.4 Settlement in soil using stress coefficients

The settlement of a pile is equal to the displacement of the pile and the elastic shortening of the pile. For end bearing piles, the pile displacement is relatively small and the main displacement is the elastic shortening of the pile. For friction piles the displacement of the pile will cause a significant settlement.

4.4.1 Elastic shortening of the pile

The elastic shortening of the pile due to elasticity of its material is given by:

$$\Delta = \frac{P\,l}{A_p E_p} \tag{4.6}$$

where:

Ep Modulus of Elasticity of the pile material, [kN/m^2].
Ap Cross-section area of the pile element, [m^2].
l Length of the pile, [m].

4.4.2 Linear elastic deformation of the soil

The linear relation between stress and strain that obeys Hooke's law is given by:

$$E = \Delta\sigma / \varepsilon \tag{4.7}$$

where:

- ε Strain in the soil layer
- $\Delta\sigma$ Average effective stress in the soil layer. If the soil layer is relatively small, $\Delta\sigma$ may be considered equal to the vertical stress in the middle of the layer σ_z.
- *E* Elastic modulus of the soil layer. Often, the 'elastic modulus' is called Young's Modulus or Modulus of Elasticity of the soil.

Young's Modulus is the modulus for when lateral deformation is allowed, which may be the case for soil loaded by a small footing, but not when loading a larger area. In the latter case, the lateral expansion is constrained. The constrained modulus, E_s, is larger than the Elastic modulus E. The constrained modulus is also called the "oedometer modulus" or Modulus of Compressibility. The inverse of Modulus of Compressibility is the Coefficient of Volume Change $m_v = 1/E_s$.

For ideally elastic soils, the ratio between E_s and E is:

$$E = E_s \frac{1 - \nu_s - 2\nu_s^{\,2}}{1 - \nu_s} \tag{4.8}$$

where:

ν_s *Poisson's* ratio of the soil. *Poisson*'s ratio in general ranges between 0 and 0.5.

Equation 4.8 shows that:

_ In the limit case ν_s = 0 (deformation without lateral strain), the values of E and E_s are equal

_ In the other limit case ν_s = 0.5 (deformation with constant volume), Modulus of Elasticity will be $E = 0 \times E_s$. In this case, only the immediate settlement (lateral deformation with constant volume) can be determined. By the other way, the first and the second terms in *Mindlin's* formula (4. 1) will be omitted, if *Poisson*'s ratio ν_s = 0.5 is used

_ For a soil material with a *Poisson's* ratio of 0.3 [-], a common value, Modulus of Compressibility E_s is 35 % larger than the Young's modulus E.

One of the several reasons for assuming that the soil consolidates in response to the change in vertical stress under a surface load is the relative simplicity of the *Boussinesq's* solution for vertical stress, which do not contain *Poisson's* ratio. For simplicity, the same equations for estimating soil settlement from stress due to a surface load can be used also here for s subsurface load. In this case, *Poisson's* ratio is assumed to be 0.3 [-].

For linear elastic deformation behavior, the settlement of a soil layer, s, of thickness, h, is given by:

$$s = \frac{1}{E_s} \Delta\sigma \; h \tag{4.9}$$

4.4.3 Non-linear elastic deformation of the soil

Stress-strain behavior is non-linear for most soils. The non-linearity cannot be disregarded when analyzing compressible soils, such as silts and clays, that is, the elastic modulus approach is not appropriate for these soils. Non-linear stress-strain behavior of compressible soils, is conventionally modeled by the following equation:

$$s = \frac{C_c\, h}{1 + e_o} \log\left(\frac{\sigma_o + \Delta\sigma}{\sigma_o} \right) \tag{4.10}$$

where:

σ_o Initial overburden pressure at the middle of the layer, $\sigma_o = \gamma z$, [kN/m^2].
$\Delta\sigma$ Average increase of stress in the layer due to the applied load, [kN/m^2].
C_c Compression index, [-].
e_o Initial void ratio, [-].

4.5 Defining the project data

4.5.1 Firm Header

When printing the results, the main data (firm name) are displayed on each page at the top in two lines. Firm name can be defined, modified and saved using the "Firm Header" option from the setting tab (see Figure 4.3).

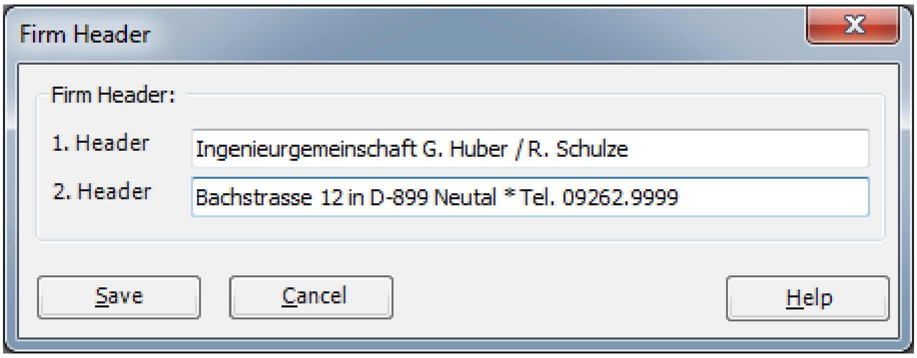

Figure 4.3 Firm Header

4.5.2 Task of the program *GEO Tools* (Analysis Type)

The program *GEO Tools* can be used to analyze various problems in Geotechnical Engineering for shallow foundations and deep foundations, Figure 4.4.

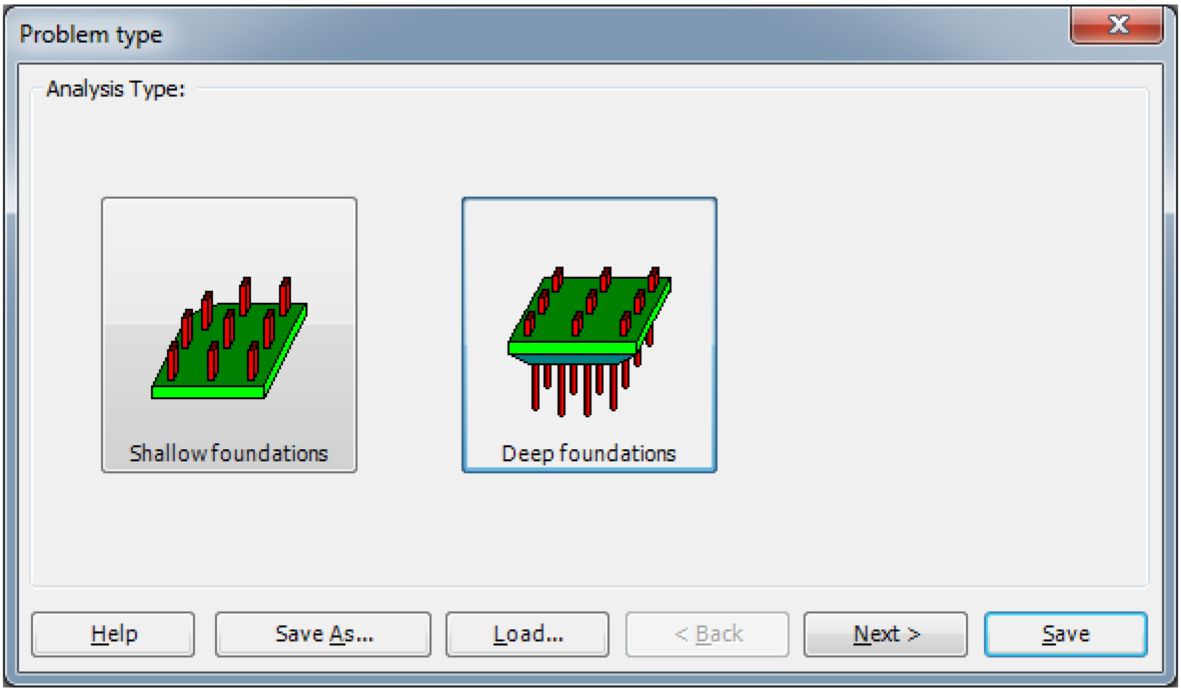

Figure 4.4 Problem type

According to the main menu shown in Figure 4.5, the following geotechnical problems can be calculated for deep foundations:

- Analysis of single pile
- Bearing capacity and settlement of single pile or pile wall
- Analysis of piled raft
- Stress coefficients according to *GEDDES*
- Sheet pile wall
- Analysis of single barrette

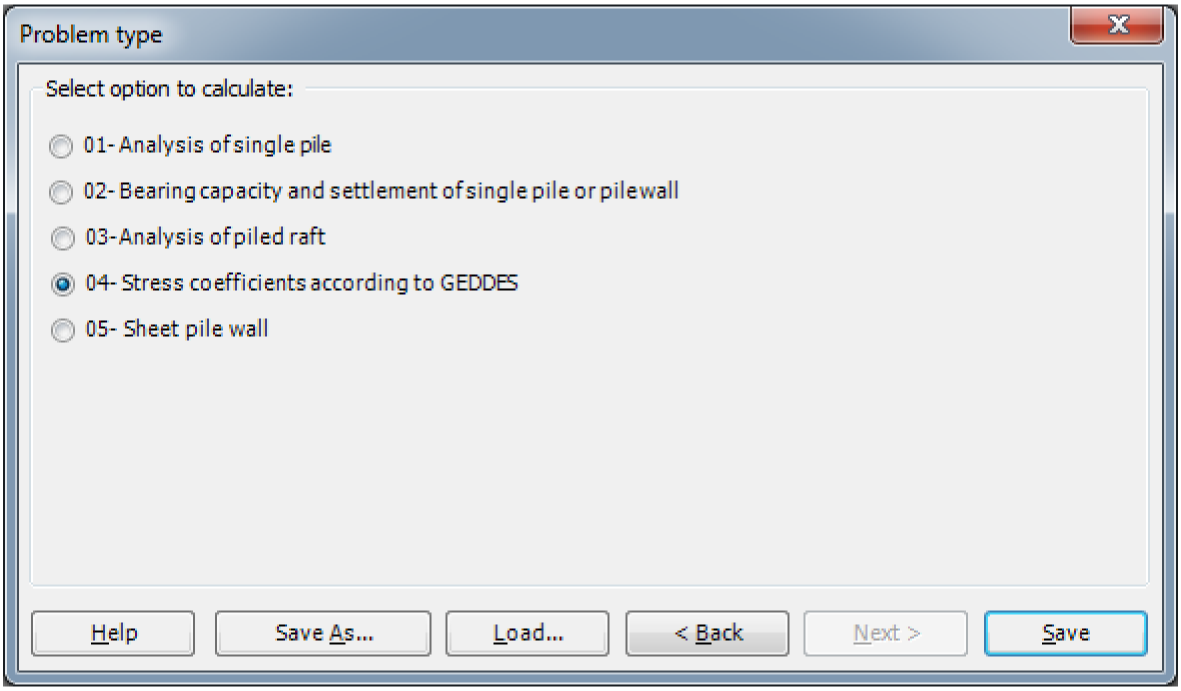

Figure 4.5 Problem type for shallow foundation

In menu of Figure 4.5 select the option:

- Stress coefficients according to *GEDDES*

The following paragraph describes how to determine stress coefficients according to *GEDDES* using the program *GEO Tools*. The input data are pile-soil system, *Poisson's* ratio, depth ratio and radial distance ratio.

4.5.3 Project Identification

In the program, it must be distinguished between the following two data groups:

1. System data (For identification of the project that is created and information to the output for the printer).

2. Soil data (Soil properties and so on).

The defining input data for these data groups is carried out as follows:

After clicking on the "Project Identification" option, the following general project data are defined (Figure 4.6):

Title: Title label
Date: Date
Project: Project label

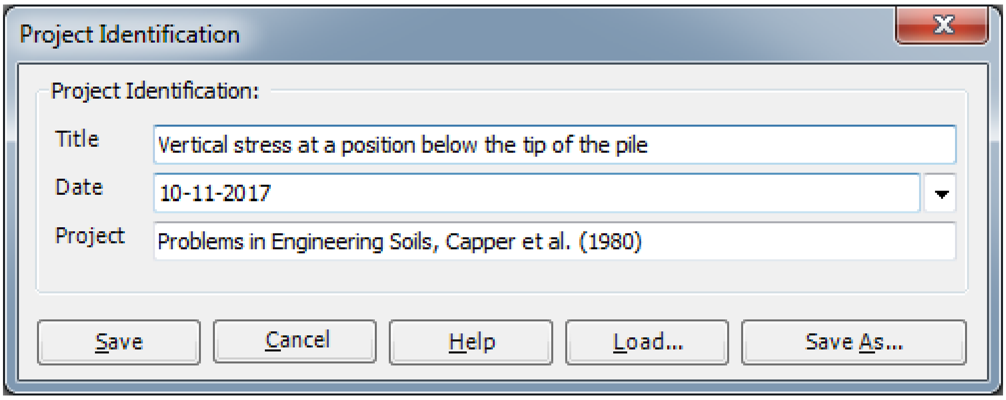

Figure 4.6 Project Identification

4.5.4 Data of stress coefficients according to *GEDDES*

After clicking on the "Stress coefficients according to *GEDDES*" option, the following data required to determine stress coefficients according to *GEDDES* are defined (Figure 4.7):

Pile-soil system:

- *Poisson's* ratio ν_s
- Stress coefficients for point (end bearing) load. *Mindlin's* case.
- Stress coefficients for uniform skin friction.
- Stress coefficients for linear variation of skin friction

Depth ratio *M=z/l*

- Start value
- No. of ratio intervals
- Ratio interval *DM* [m]

Radial distance ratio *N=r/l*

- Start value
- No. of ratio intervals
- Ratio interval *DN* [m]

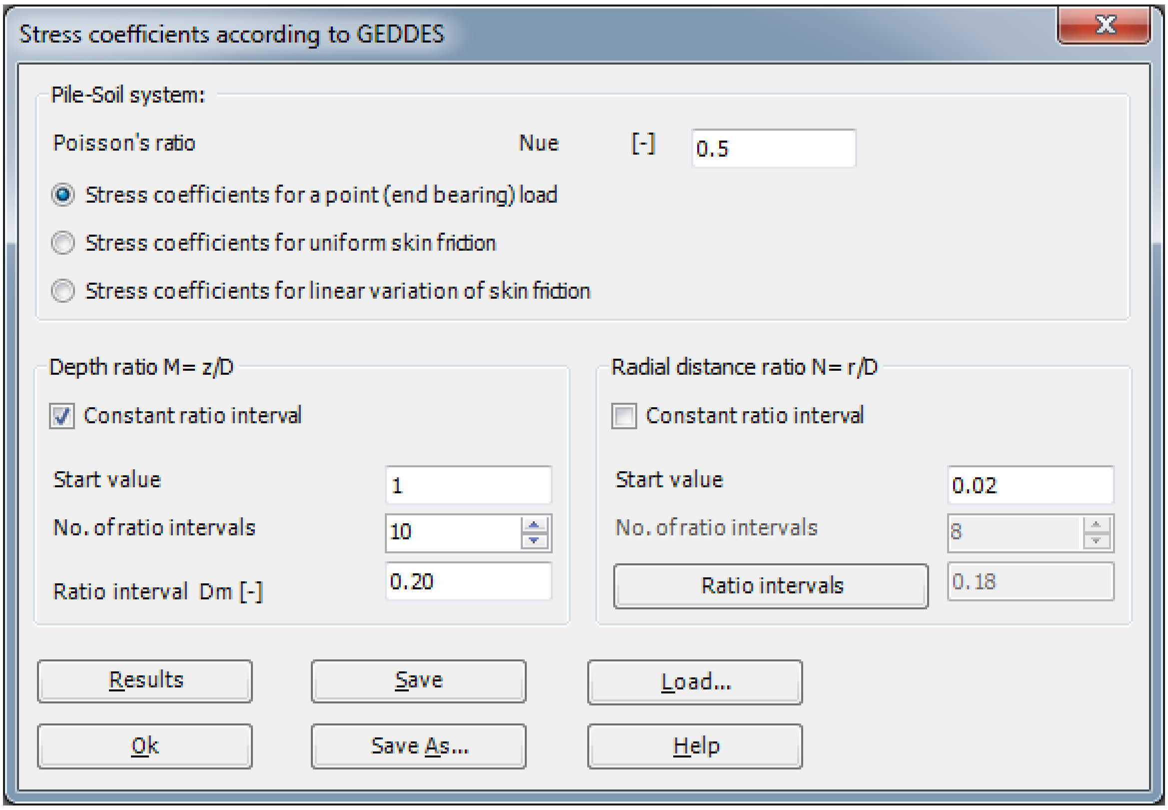

Figure 4.7 Stress coefficients according to *GEDDES*

4.6 Examples to verify stress coefficients according to *GEDDES*

To verify and evaluate the stress coefficients according to *GEDDES*, a series of comparison were carried out in which the stress coefficients according to *GEDDES* obtained by *GEO Tools* were compared with those from existing references.

4.6.1 Example 1: Vertical stress at a position below the tip of the pile

4.6.1.1 Description of the problem

To verify the vertical stress at point below the tip of a pile, the stress coefficients K_z for a point (end bearing) and uniform skin friction below the tip of the pile obtained by *Capper et al.* (1980), Table 8.13A in page 230 and Table 8.14 in page 232 are compared with those obtained by *GEO Tools*.

Figure 4.8 shows a vertical pile l=10 [m] long carries a load of P=1 [MN] of which 400 [kN] is carried by point bearing and the remainder by side friction. The material in which the pile is embedded can be assumed to have a *Poisson's* ratio of ν_s=0.5 [-]. It is required to determine the vertical stress at a position d=6.7 [m] below the tip of the pile at a horizontal radial distance of r=2 [m]. The friction force is uniformly distributed along the length of the pile.

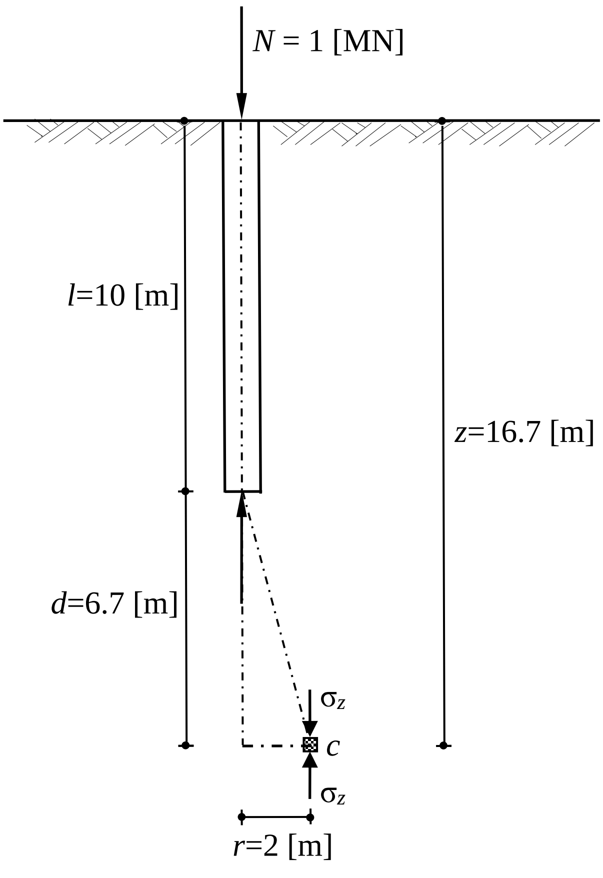

Figure 4.8 Vertical pile with dimension

4.6.1.2 Hand calculation of stress in soil

According to *Capper et al.* (1980), the vertical stress σ_z [kN/m^2] at a position d=6.7 [m] below the tip of the pile at a horizontal radial distance of r=2 [m] can be calculated as follows:

Radial distance and vertical depths are:

$$r = 2 \ \text{[m]}$$
$$l = 10 \ \text{[m]}$$
$$z = 10 + 6.7 = 16.7 \ \text{[m]}$$

Depth and radial distance ratios:

$$M = \frac{z}{l} = \frac{16.7}{10} = 1.67 \ [\text{-}]$$
$$N = \frac{r}{l} = \frac{2}{10} = 0.2 \ [\text{-}]$$

From Table 8.13A in page 230 of the reference *Capper et al.* (1980), the stress coefficient due to a point load is:

$$K_z = 0.497 \ [\text{-}]$$

The stress due to the point load is:

$$\sigma_{z1} = \frac{p}{l^2} K_z = \frac{400}{10^2} 0.497 = 2 \ [\text{kN/m}^2]$$

From Table 8.14 in page 232 of the reference *Capper et al.* (1980), the stress coefficient due to the uniformly distributed friction load is:

$$K_z = 0.273 \ [\text{-}]$$

The stress due to the uniformly distributed friction load is:

$$\sigma_{z2} = \frac{p}{l^2} K_z = \frac{(1000 - 400)}{10^2} 0.273 = 1.6 \, [\text{kN/m}^2]$$

The total vertical is:

$$\sigma_{zt} = \sigma_{z1} + \sigma_{z2} = 2 + 1.6 = 3.6 \, [\text{kN/m}^2]$$

Table 4.1 Influence factors for vertical pressure caused by a vertical point load acting below the surface at a depth *l*. *Poisson's* ratio ν_s=0.5 [-] (Table 8.13A in the reference *Capper et al.* (1980))

M=z/l \ N=r/l	0.0	0.2	0.4	0.6	1.0	1.4	2.0	2.5	3.0
1.0	-	0.115	0.103	0.086	0.051	0.026	0.008	0.003	0.001
1.2	6.067	0.150	0.194	0.094	0.050	0.027	0.010	0.004	0.001
1.4	1.574	0.934	0.338	0.144	0.055	0.029	0.011	0.005	0.002
1.6	0.732	0.577	0.328	0.174	0.065	0.033	0.013	0.006	0.003
1.8	0.431	0.378	0.268	0.172	0.073	0.037	0.015	0.007	0.004
2.0	0.289	0.266	0.212	0.154	0.076	0.040	0.017	0.009	0.004
2.2	0.209	0.197	0.168	0.133	0.075	0.042	0.019	0.010	0.005
2.4	0.160	0.153	0.136	0.114	0.071	0.043	0.020	0.011	0.006
2.6	0.126	0.123	0.112	0.097	0.066	0.042	0.021	0.012	0.007
2.8	0.103	0.101	0.093	0.083	0.061	0.041	0.022	0.013	0.008
3.0	0.086	0.084	0.079	0.072	0.055	0.039	0.022	0.014	0.008

To obtain the vertical pressure at a given depth *z* multiply the appropriate factor for the desired *z/l* ratio and radial distance ratio *r/l* by the total value of the point load *P* and divide by l^2.

Table 4.2 Influence factors for vertical pressure caused by a uniformly distributed vertical line load acting below the surface at a depth *l*. *Poisson's* ratio ν_s=0.5[-] (Table 8.14 in the reference *Capper et al.* (1980))

M=z/l \ N=r/l	0.0	0.2	0.4	0.6	1.0	1.4	2.0	2.5	3.0
1.0	-	0.750	0.337	0.189	0.067	0.025	0.006	0.002	0.001
1.2	1.420	0.649	0.329	0.193	0.076	0.032	0.009	0.003	0.001
1.4	0.540	0.440	0.289	0.186	0.082	0.037	0.012	.0005	0.002
1.6	0.339	0.305	0.235	0.169	0.084	0.041	0.015	0.006	0.003
1.8	0.239	0.224	0.188	0.147	0.082	0.044	0.017	0.008	0.004
2.0	0.180	0.172	0.151	0.125	0.077	0.045	0.019	0.010	0.005
2.2	0.141	0.136	0.124	0.107	0.072	0.045	0.021	0.011	0.006
2.4	0.114	0.111	0.103	0.091	0.065	0.043	0.022	0.012	0.007
2.6	0.094	0.093	0.087	0.079	0.059	0.041	0.022	0.013	0.008
2.8	0.080	0.078	0.074	0.068	0.054	0.039	0.023	0.014	0.008
3.0	0.068	0.067	0.064	0.060	0.049	0.037	0.022	0.014	0.009

To obtain the vertical pressure at a given depth *z* multiply the appropriate factor for the desired *z/l* ratio and radial distance ratio *r/l* by the total value of the distributed line load P_f and divide by l^2.

4.6.1.3 Stress coefficients according to GEDDES by GEO Tools

The location of the stress in soil under the pile can be defined at any position in *GEO Tools*. The stress coefficients obtained by *GEO Tools* required for this example are presented on the next pages in tabulation form and diagrams.

Table 4.1 [Table 8.13A in page 230 of the reference *Capper et al.* (1980)] shows the stress coefficient due to a point load, while Table 4.2 [Table 8.14 in page 232 of the reference *Capper et al.* (1980)] shows the stress coefficient due to the uniformly distributed friction load. From these tables, it can be observed that the stress coefficients obtained by *GEO Tools* are equal to those obtained by *Capper et al.* (1980).

```
*************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
*************************************************************
Title: Vertical stress at a position below the tip of the pile
Date: 10-11-2017
Project: Problems in Engineering Soils, Capper et al. (1980)
File: Capper et al. (1980)

-------------------------------------------------------------
Stress coefficients according to GEDDES
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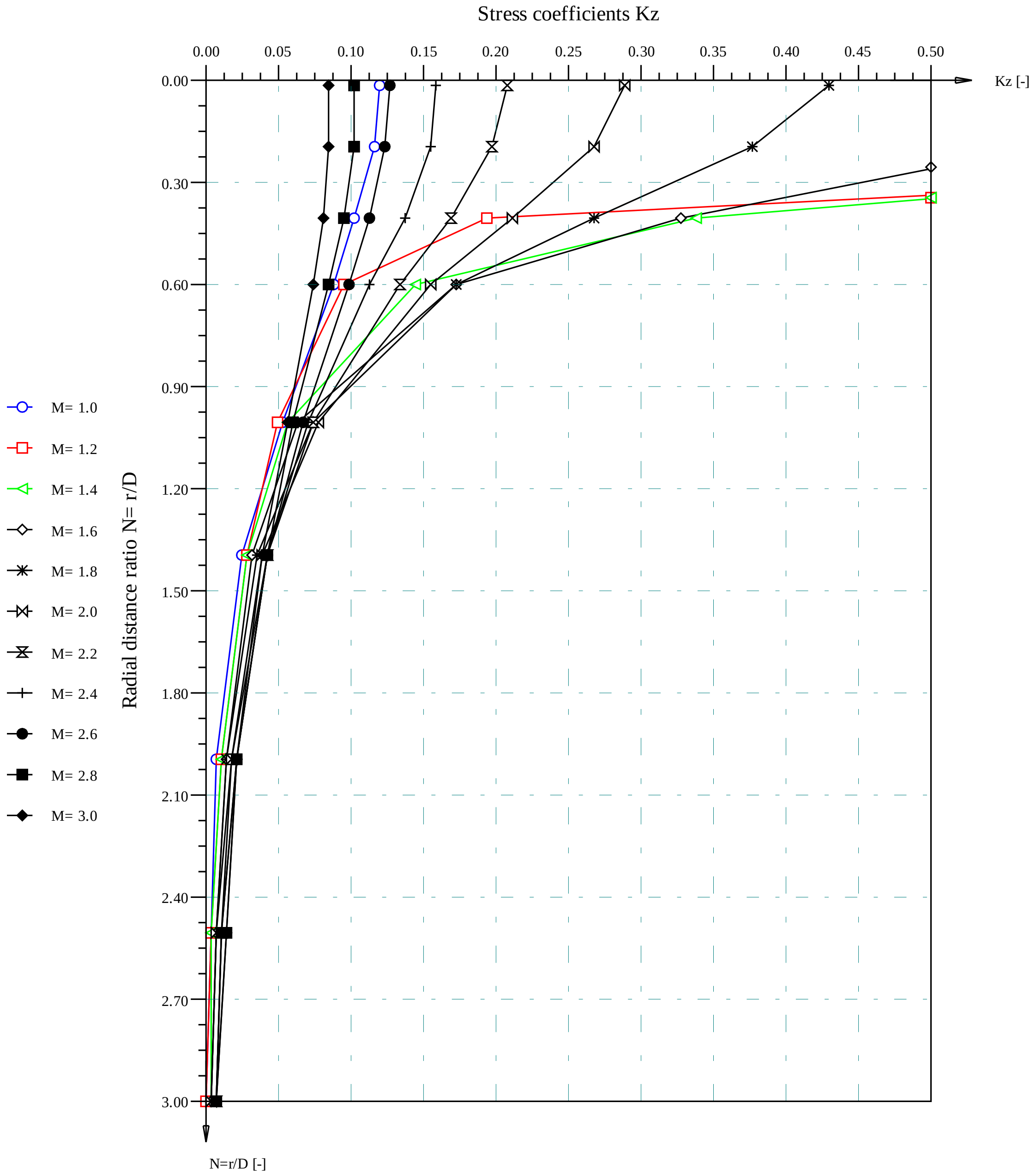
Stress coefficients for a point (end bearing) load,
Poisson's ratio = 0.50 [-]:

M/N	0.02	0.20	0.40	0.60	1.00	1.40	2.00	2.50	3.00
1.0	0.1193	0.1150	0.1030	0.0863	0.0512	0.0259	0.0079	0.0027	0.0008
1.2	5.9199	1.1503	0.1937	0.0939	0.0501	0.0273	0.0096	0.0037	0.0014
1.4	1.5645	0.9338	0.3376	0.1435	0.0553	0.0293	0.0113	0.0048	0.0020
1.6	0.7300	0.5768	0.3275	0.1739	0.0650	0.0327	0.0131	0.0060	0.0027
1.8	0.4308	0.3779	0.2677	0.1717	0.0729	0.0366	0.0151	0.0073	0.0035
2.0	0.2886	0.2657	0.2117	0.1540	0.0762	0.0400	0.0170	0.0086	0.0044
2.2	0.2090	0.1975	0.1684	0.1331	0.0752	0.0421	0.0188	0.0099	0.0053
2.4	0.1595	0.1531	0.1361	0.1137	0.0714	0.0429	0.0203	0.0111	0.0062
2.6	0.1264	0.1225	0.1119	0.0972	0.0663	0.0424	0.0214	0.0122	0.0070
2.8	0.1030	0.1005	0.0935	0.0835	0.0607	0.0411	0.0220	0.0130	0.0077
3.0	0.0858	0.0841	0.0793	0.0722	0.0552	0.0393	0.0223	0.0137	0.0084

Stress coefficients for uniform skin friction,
Poisson's ratio = 0.50 [-]:

M/N	0.02	0.20	0.40	0.60	1.00	1.40	2.00	2.50	3.00
1.0	7.9181	0.7496	0.3369	0.1888	0.0674	0.0249	0.0059	0.0019	0.0006
1.2	1.1327	0.6489	0.3290	0.1926	0.0762	0.0315	0.0088	0.0032	0.0012
1.4	0.5387	0.4398	0.2887	0.1862	0.0818	0.0372	0.0118	0.0048	0.0020
1.6	0.3387	0.3049	0.2345	0.1685	0.0835	0.0414	0.0147	0.0064	0.0029
1.8	0.2386	0.2238	0.1875	0.1465	0.0817	0.0439	0.0173	0.0081	0.0039
2.0	0.1793	0.1718	0.1513	0.1252	0.0773	0.0449	0.0193	0.0097	0.0049
2.2	0.1408	0.1364	0.1239	0.1068	0.0716	0.0446	0.0208	0.0110	0.0059
2.4	0.1143	0.1112	0.1030	0.0914	0.0654	0.0433	0.0218	0.0122	0.0068
2.6	0.0947	0.0925	0.0869	0.0788	0.0594	0.0414	0.0223	0.0131	0.0076
2.8	0.0797	0.0782	0.0743	0.0684	0.0537	0.0391	0.0225	0.0137	0.0083
3.0	0.0684	0.0670	0.0641	0.0598	0.0485	0.0367	0.0222	0.0141	0.0089

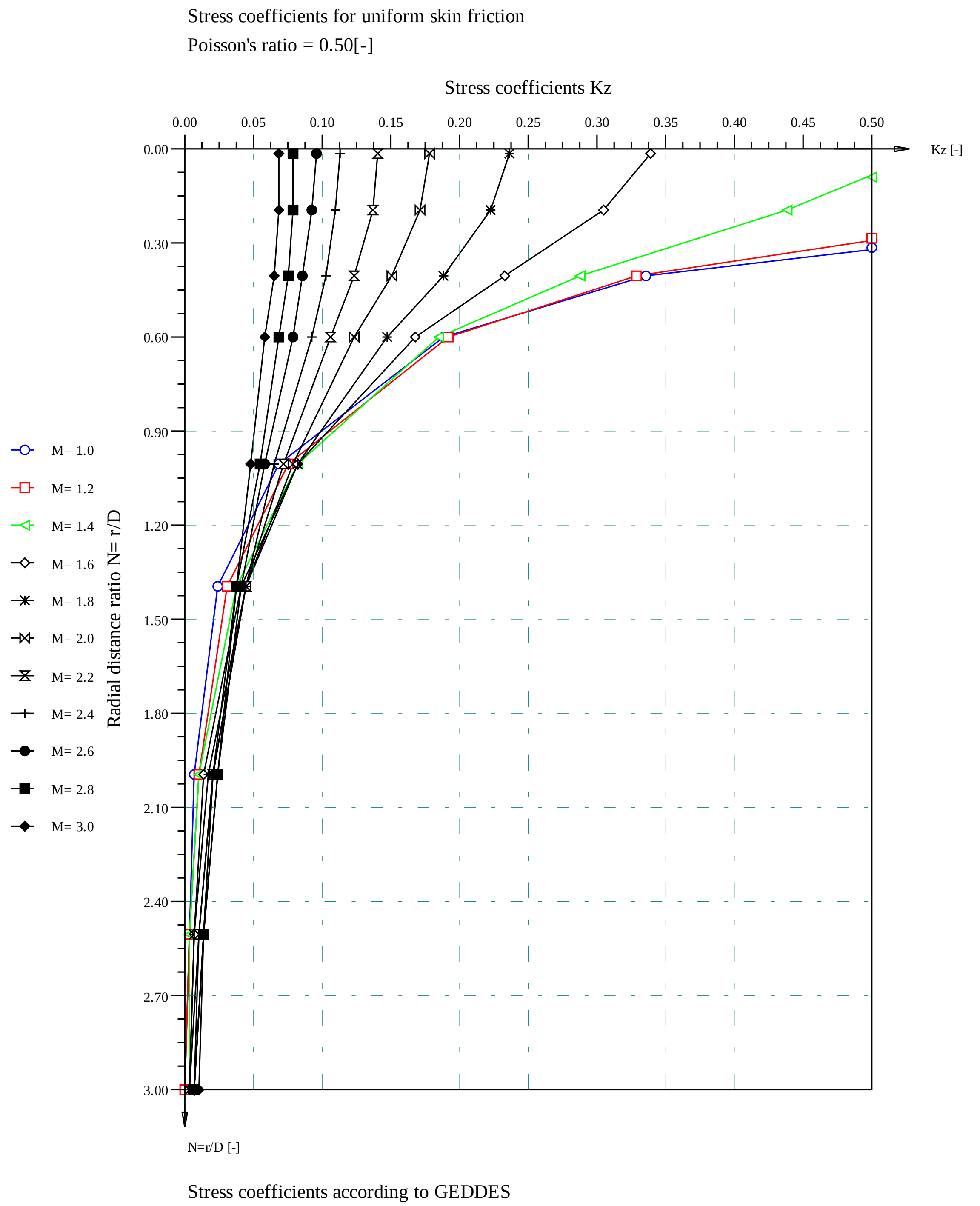
Stress coefficients for a point (end bearing) load
Poisson's ratio = 0.50[-]



Stress coefficients according to GEDDES

GEOTEC Software Inc., Calgary AB, Canada T3E 7Y7
PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323

Scale: 4	Project: Problems in Engineering Soils, Capper et al. (1980)
File: Capper et al. (1980)	Date: 10-11-2017
Page No.:	Title: Vertical stress at a position below the tip of the pile



Stress coefficients according to GEDDES

GEOTEC Software Inc., Calgary AB, Canada T3E 7Y7 PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323	
Scale: 3	Project: Problems in Engineering Soils, Capper et al. (1980)
File: Capper et al. (1980)	Date: 10-11-2017
Page No.:	Title: Vertical stress at a position below the tip of the pile

4.6.2 Example 2: Vertical stress at a point of the four-pile group

4.6.2.1 Description of the problem

To verify the vertical stress at point below a free standing pile cap, the stress coefficients K_z for a point (end bearing) and uniform skin friction below the center of a four-pile group obtained by *Bowels* (1997), Example 18-3, page 1015, are compared with those obtained by *GEO Tools*.

Determine the vertical stress under the center *A* of the square pile cape shown in Figure 4.9. The pile cap is rested on four-pile group and loaded by a central vertical load of N = 2000 [kN]. The vertical stress is required at a depth of 1.5 [m] from the pile tips as shown in Figure 4.9. Piles have a length of l = 16.8 [m]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume the following two cases:

- Assume point-loaded piles.
- Assume one-half of load carried by point and one-half carried by friction.

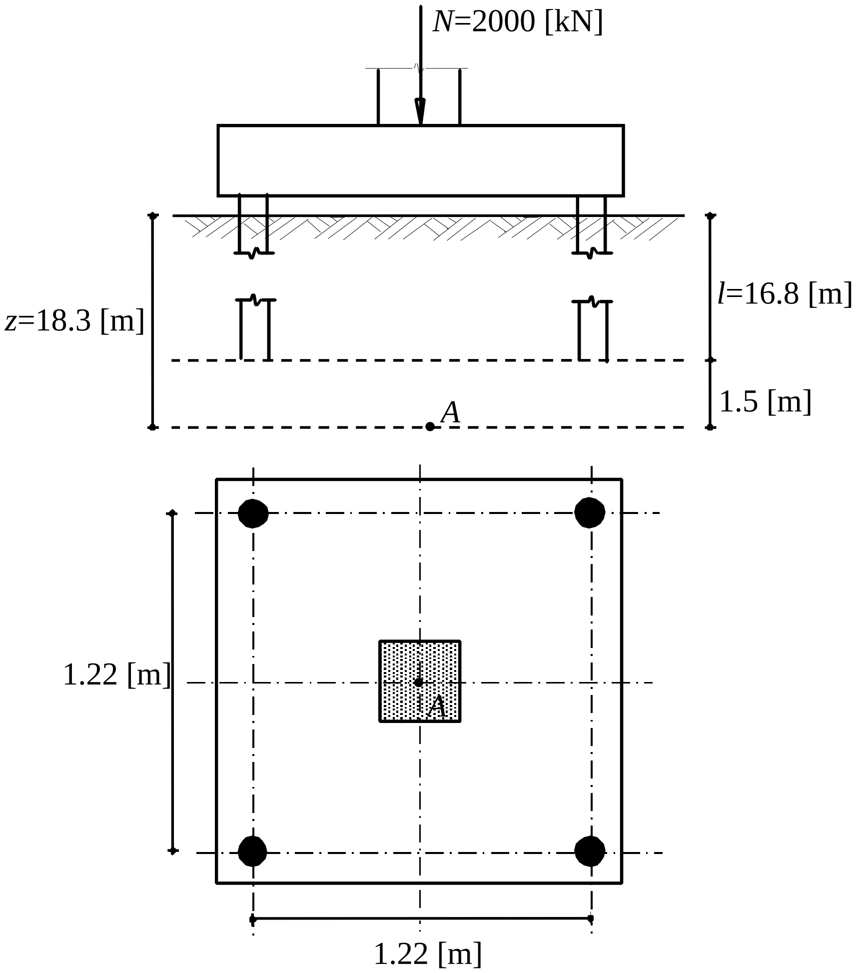


Figure 4.9 Pile cap with piles

4.6.2.2 Hand calculation

4.6.2.2.1 Assume point-loaded piles

Force in a single pile:

$$P_p = \frac{2000}{4} = 500 \ \text{[kN]}$$

Depth and radial distance ratios:

$$z = 16.8 + 1.5 = 18.3 \ \text{[m]}$$

$$r = 0.610\sqrt{2} = 0.863 \ \text{[m]}$$

$$M = \frac{z}{l} = \frac{18.3}{16.8} = 1.09 \ \text{[-]}$$

$$N = \frac{r}{l} = \frac{0.863}{16.8} = 0.0514 \ \text{[-]}$$

From Table 18-1a with: r = 0.863, l = 16.8, *Poission's* ratio ν_s = 0.3 [-], case 1, z = 18.3. The stress coefficient is:

$$K_z = 12.41 \ \text{[-]}$$

Stress at point *A*:

$$\sigma_z = \frac{4p}{l^2} K_z = \frac{4 \times 500}{16.8^2} 12.41 = 87.87 \ [\text{kN/m}^2]$$

4.6.2.2.2 Assume one-half of load carried by point and one-half carried by friction

As in result 1 preceding. For point,

$$\sigma_{z1} = \frac{4p}{l^2} K_z = \frac{1}{2} \frac{4 \times 500}{16.8^2} 12.41 = 43.94 \ [\text{kN/m}^2]$$

For constant variation of skin friction (case 2), the stress coefficient is:

$$K_z = 1.73 \ \text{[-]}$$

$$\sigma_{z2} = \frac{4p}{l^2} K_z = \frac{1}{2} \frac{4 \times 500}{16.8^2} 1.73 = 6.13 \ [\text{kN/m}^2]$$

Total stress at point *A*:

$$\sigma_{z(\text{total})} = 43.94 + 6.13 = 50 \ [\text{kN/m}^2]$$

Table 4.3 to Table 4.5 [originally Table 18-1 in the reference *Bowels* (1997)] list values for K_z for various $M = z/l$ and $N = r/l$ values and three selected values of *Poisson's* ratio ν_s for all three cases. By superposition of effects, these three cases should provide a general solution for the vertical stress at a point for any reasonable type of stress distribution along a pile. To avoid interpolation use the program *GEO Tools* for any of these 3 skin resistance cases.

Values are not shown in Table 4.3 to Table 4.5 for any $M = z/l < 1.00$ for any cases 1-3, as these represent a tension stress in the soil at depth z above the pile tip. Soil cannot resist tensile stress. Only compressive soil stresses in the strata below the soil tip cause settlement, although the pile cap settlement is actually the sum of both point settlement and elastic axial pile shaft deformation. Also it was necessary to use N = 0.002+ when programming the case 2 and case 3 table output, since N = 0.0 would produce a discontinuity (divide-by-zero error) in the computations.

4.6.2.3 Stress coefficients according to GEDDES by GEO Tools

The location of the stress in soil under the pile can be defined at any position in *GEO Tools*. The stress coefficients obtained by *GEO Tools* required for this example are presented on the next pages in tabulation form and diagrams. From these tables, it can be observed that the stress coefficients obtained by *GEO Tools* are equal to those obtained by *Bowels* (1997).

Table 4.3 Stress coefficients for a point load as shown in case 1 of Figure 4.2

M = z/l; N = r/l

[TABLE 18-1a in *Bowels* (1997)]

M	**N =0.0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.75**	**1.0**	**1.5**	**2.0**
Poisson's ratio ν_s=0.20										
1.0		0.0960	0.0936	0.0897	0.0846	0.0785	0.0614	0.0448	0.0208	0.0089
1.1	17.969	3.7753	0.6188	0.2238	0.1332	0.0999	0.0659	0.0467	0.0222	0.0099
1.2	4.5510	2.7458	1.0005	0.3987	0.2056	0.1325	0.0724	0.0490	0.0236	0.0110
1.3	2.0609	1.6287	0.9233	0.4798	0.2672	0.1681	0.0811	0.0520	0.0249	0.0119
1.4	1.1858	1.0328	0.7330	0.4652	0.2926	0.1930	0.0905	0.0555	0.0263	0.0129
1.5	0.7782	0.7153	0.5682	0.4114	0.2875	0.2025	0.0985	0.0592	0.0277	0.0138
1.6	0.5548	0.5238	0.4457	0.3518	0.2664	0.1997	0.1038	0.0625	0.0290	0.0147
1.7	0.4188	0.4018	0.3569	0.2984	0.2399	0.1893	0.1061	0.0651	0.0303	0.0156
1.8	0.3294	0.3193	0.2918	0.2539	0.2133	0.1755	0.1057	0.0668	0.0315	0.0164
1.9	0.2673	0.2609	0.2431	0.2177	0.1890	0.1606	0.1033	0.0675	0.0325	0.0172
2.0	0.2222	0.2180	0.2060	0.1883	0.1676	0.1462	0.0995	0.0673	0.0334	0.0179
Poisson's ratio ν_s=0.30										
1.0		0.101	0.0986	0.0944	0.0889	0.0824	0.0641	0.0463	0.0209	0.0087
1.1	19.393	3.9054	0.5978	0.2123	0.1287	0.0986	0.0668	0.0475	0.0222	0.0097
1.2	4.9099	2.9275	1.0358	0.4001	0.2027	0.1303	0.0722	0.0493	0.0235	0.0106
1.3	2.2222	1.7467	0.9757	0.4970	0.2717	0.1687	0.0808	0.0519	0.0247	0.0116
1.4	1.2777	1.1152	0.7805	0.4891	0.3032	0.1974	0.0908	0.0555	0.0260	0.0125
1.5	0.8377	0.7686	0.6070	0.4356	0.3012	0.2098	0.0999	0.0594	0.0274	0.0134
1.6	0.598	0.5626	0.4768	0.3738	0.2809	0.2086	0.1063	0.0631	0.0288	0.0143
1.7	0.4500	0.4312	0.3819	0.3177	0.2538	0.1988	0.1094	0.0661	0.0302	0.0152
1.8	0.3536	0.3424	0.3122	0.2706	0.2262	0.1849	0.1096	0.0682	0.0315	0.0161
1.9	0.2866	0.2795	0.2600	0.2321	0.2006	0.1697	0.1076	0.0693	0.0326	0.0169
2.0	0.2380	0.2333	0.2201	0.2007	0.1780	0.1547	0.1039	0.0694	0.0336	0.0177
Poisson's ratio ν_s=0.40										
1.0		0.1083	0.1054	0.1008	0.0947	0.0876	0.0676	0.0483	0.0212	0.0083
1.1	21.291	4.0788	0.5699	0.1970	0.1228	0.0970	0.0680	0.0486	0.0223	0.0093
1.2	5.3884	3.1699	1.0829	0.4020	0.1989	0.1274	0.0720	0.0496	0.0233	0.0102
1.3	2.4373	1.9040	1.0455	0.5200	0.2776	0.1695	0.0804	0.0519	0.0244	0.0111
1.4	1.4002	1.2179	0.8438	0.5208	0.3173	0.2032	0.0913	0.0554	0.0256	0.0120
1.5	0.9172	0.8395	0.6587	0.4678	0.3194	0.2196	0.1017	0.0596	0.0270	0.0129
1.6	0.6527	0.6143	0.5181	0.4033	0.3001	0.2205	0.1095	0.0638	0.0284	0.0138
1.7	0.4915	0.4705	0.4152	0.3435	0.2724	0.2116	0.1138	0.0675	0.0300	0.0147
1.8	0.3858	0.3732	0.3393	0.2929	0.2433	0.1976	0.1148	0.0701	0.0314	0.0156
1.9	0.3123	0.3044	0.2825	0.2512	0.2161	0.1818	0.1133	0.0717	0.0328	0.0166
2.0	0.2590	0.2537	0.2390	0.2173	0.1919	0.1659	0.1098	0.0722	0.0340	0.0174

Table 4.4 Stress coefficients for constant skin friction as shown in case 2 of Figure 4.2
M = z/l; N = r/l
[TABLE 18-1b in *Bowels* (1997)]

M	***N* = 0.0**	**0.02**	**0.04**	**0.06**	**0.08**	**0.10**	**0.15**	**0.20**	**0.50**	**1.0**	**2.0**
Poisson's ratio ν_s=0.20											
1.0		6.4703	3.2374	2.1592	1.6202	1.2962	0.8630	0.6445	0.2300	0.0690	0.0081
1.1	1.7781	1.7342	1.5944	1.4178	1.2418	1.0850	0.7953	0.6138	0.2283	0.0730	0.0096
1.2	0.9015	0.8789	0.8576	0.8269	0.7882	0.7446	0.6317	0.5307	0.2231	0.0759	0.0111
1.3	0.5968	0.5799	0.5725	0.5629	0.5500	0.5340	0.4867	0.4355	0.2138	0.0779	0.0125
1.4	0.4569	0.4288	0.4241	0.4201	0.4142	0.4068	0.3838	0.3562	0.2010	0.0789	0.0139
1.5	0.3482	0.3359	0.3334	0.3313	0.3282	0.3242	0.3113	0.2952	0.1862	0.0790	0.0152
1.6	0.2922	0.2726	0.2716	0.2707	0.2689	0.2666	0.2589	0.2487	0.1708	0.0784	0.0165
1.7	0.2518	0.2304	0.2287	0.2274	0.2261	0.2247	0.2195	0.2127	0.1559	0.0770	0.0175
1.8	0.1772	0.1953	0.1949	0.1942	0.1936	0.1925	0.1891	0.1844	0.1420	0.0750	0.0185
1.9	0.1648	0.1702	0.1698	0.1687	0.1682	0.1675	0.1650	0.1616	0.1295	0.0727	0.0193
2.0	0.1461	0.1482	0.1486	0.1480	0.1478	0.1473	0.1455	0.1429	0.1180	0.0700	0.0201
Poisson's ratio ν_s=0.30											
1.0		6.8419	3.4044	2.2673	1.6983	1.3567	0.8998	0.6695	0.2346	0.0686	0.0076
1.1	1.9219	1.8611	1.7072	1.5134	1.3211	1.1503	0.8368	0.6419	0.2335	0.0728	0.0091
1.2	0.9699	0.9403	0.9166	0.8825	0.8400	0.7922	0.6688	0.5588	0.2292	0.0760	0.0105
1.3	0.6430	0.6188	0.6099	0.5992	0.5850	0.5675	0.5157	0.4597	0.2207	0.0782	0.0120
1.4	0.4867	0.4558	0.4507	0.4461	0.4396	0.4316	0.4063	0.3761	0.2082	0.0796	0.0134
1.5	0.3766	0.3561	0.3533	0.3510	0.3476	0.3432	0.3291	0.3115	0.1834	0.0800	0.0148
1.6	0.3339	0.2895	0.2878	0.2863	0.2843	0.2817	0.2732	0.2621	0.1777	0.0796	0.0160
1.7	0.2664	0.2438	0.2414	0.2399	0.2384	0.2369	0.2313	0.2239	0.1623	0.0784	0.0172
1.8	0.2025	0.2065	0.2054	0.2044	0.2038	0.2026	0.1989	0.1938	0.1479	0.0766	0.0182
1.9	0.1847	0.1794	0.1785	0.1777	0.1768	0.1760	0.1733	0.1696	0.1347	0.0744	0.0191
2.0	0.1634	0.1565	0.1561	0.1556	0.1551	0.1545	0.1525	0.1498	0.1229	0.0718	0.0199
Poisson's ratio ν_s=0.40											
1.0		7.2744	3.6270	2.4110	1.8026	1.4373	0.9488	0.7029	0.2407	0.0681	0.0069
1.1	2.0931	2.0296	1.8574	1.6409	1.4266	1.2372	0.8921	0.6794	0.2404	0.0725	0.0083
1.2	1.0486	1.0209	0.9947	0.9567	0.9091	0.8556	0.7181	0.5964	0.2373	0.0760	0.0098
1.3	0.6922	0.6694	0.6598	0.6476	0.6318	0.6122	0.5543	0.4921	0.2298	0.0787	0.0113
1.4	0.5347	0.4922	0.4860	0.4807	0.4735	0.4645	0.4362	0.4026	0.2178	0.0805	0.0128
1.5	0.4020	0.3823	0.3798	0.3771	0.3734	0.3684	0.3527	0.3332	0.2029	0.0813	0.0142
1.6	0.3440	0.3096	0.3083	0.3068	0.3045	0.3017	0.2922	0.2800	0.1868	0.0812	0.0155
1.7	0.2943	0.2606	0.2580	0.2564	0.2549	0.2531	0.2469	0.2387	0.1708	0.0803	0.0167
1.8	0.2114	0.2207	0.2189	0.2181	0.2174	0.2161	0.2119	0.2063	0.1558	0.0787	0.0178
1.9	0.1782	0.1907	0.1904	0.1890	0.1881	0.1873	0.1843	0.1802	0.1419	0.0766	0.0188
2.0	0.1741	0.1660	0.1658	0.1652	0.1648	0.1642	0.1620	0.1590	0.1294	0.0741	0.0196

Table 4.5 Stress coefficients for a linear variation of skin friction as shown in case 3 of Figure 4.2
M = z/l; M = r/l
[TABLE 18-lc in *Bowels* (1997)]

M	***N* = 0.0**	**0.02**	**0.04**	**0.06**	**0.08**	**0.10**	**0.15**	**0.20**	**0.50**	**1.0**	**2.0**
Poisson's ratio ν_s=0.20											
1.0		11.5315	5.3127	3.3023	2.3263	1.7582	1.0372	0.7033	0.1963	0.0618	0.0082
1.1	2.8427	2.7518	2.4908	2.1596	1.8329	1.5469	1.0359	0.7346	0.2074	0.0656	0.0096
1.2	1.2853	1.2541	1.2158	1.1620	1.0930	1.0162	0.8211	0.6529	0.2141	0.0689	0.0110
1.3	0.7673	0.7753	0.7585	0.7420	0.7195	0.6928	0.6142	0.5312	0.2139	0.0717	0.0123
1.4	0.5937	0.5450	0.5343	0.5267	0.5181	0.5063	0.4693	0.4261	0.2068	0.0737	0.0136
1.5	0.4485	0.4051	0.4059	0.4006	0.3960	0.3901	0.3704	0.3460	0.1947	0.0750	0.0148
1.6	0.3635	0.3201	0.2326	0.3183	0.3154	0.3123	0.3008	0.2861	0.1803	0.0754	0.0160
1.7	0.3204	0.2583	0.2635	0.2618	0.2595	0.2574	0.2503	0.2408	0.1652	0.0750	0.0170
1.8	0.2533	0.2222	0.2239	0.2206	0.2181	0.2166	0.2122	0.2059	0.1506	0.0739	0.0180
1.9	0.2382	0.1761	0.1855	0.1880	0.1878	0.1853	0.1827	0.1782	0.1371	0.0722	0.0188
2.0	0.1767	0.1643	0.1648	0.1630	0.1631	0.1614	0.1591	0.1561	0.1248	0.0700	0.0196
Poisson's ratio ν_s=0.30											
1.0		12.1310	5.5765	3.4591	2.4320	1.8346	1.0774	0.7276	0.1997	0.0616	0.0777
1.1	3.0612	2.9620	2.6751	2.3119	1.9547	1.6433	1.0908	0.7680	0.2115	0.0654	0.0090
1.2	1.3821	1.3465	1.3052	1.2465	1.1706	1.0864	0.8730	0.6899	0.2198	0.0689	0.0104
1.3	0.8262	0.8035	0.8130	0.7949	0.7705	0.7411	0.6548	0.5639	0.2212	0.0720	0.0117
1.4	0.6194	0.5827	0.5722	0.5630	0.5540	0.5410	0.5005	0.4530	0.2150	0.0744	0.0130
1.5	0.5189	0.4337	0.4332	0.4281	0.4227	0.4163	0.3946	0.3679	0.2033	0.0760	0.0143
1.6	0.3841	0.3415	0.3449	0.3395	0.3361	0.3327	0.3202	0.3039	0.1887	0.0768	0.0155
1.7	0.3332	0.2764	0.2810	0.2782	0.2764	0.2739	0.2660	0.2556	0.1732	0.0767	0.0166
1.8	0.2837	0.2268	0.2381	0.2347	0.2319	0.2300	0.2253	0.2183	0.1580	0.0758	0.0176
1.9	0.2654	0.1873	0.1963	0.1991	0.1988	0.1965	0.1937	0.1887	0.1439	0.0742	0.0186
2.0	0.1872	0.1730	0.1744	0.1732	0.1725	0.1714	0.1684	0.1651	0.1310	0.0721	0.0194
Poisson's ratio ν_s=0.40											
1.0		12.9304	5.9282	3.6683	2.5729	1.9365	1.1311	0.7600	0.2042	0.0614	0.0069
1.1	3.3525	3.2423	2.9209	2.5144	2.1171	1.7719	1.1641	0.8125	0.2170	0.0652	0.0083
1.2	1.5030	1.4712	1.4255	1.3588	1.2742	1.1800	0.9422	0.7394	0.2274	0.0689	0.0096
1.3	0.8965	0.9066	0.8862	0.8649	0.8383	0.8056	0.7089	0.6076	0.2308	0.0723	0.0109
1.4	0.6753	0.6350	0.6222	0.6120	0.6018	0.5874	0.5419	0.4890	0.2260	0.0752	0.0123
1.5	0.5629	0.4718	0.4712	0.4641	0.4584	0.4511	0.4270	0.3971	0.2147	0.0773	0.0136
1.6	0.4198	0.3701	0.3730	0.3672	0.3642	0.3600	0.3461	0.3278	0.1999	0.0786	0.0149
1.7	0.3752	0.2840	0.3039	0.3011	0.2984	0.2956	0.2870	0.2754	0.1838	0.0788	0.0161
1.8	0.3158	0.2496	0.2575	0.2530	0.2497	0.2479	0.2427	0.2349	0.1680	0.0782	0.0172
1.9	0.2851	0.2022	0.2122	0.2155	0.2142	0.2113	0.2083	0.2028	0.1530	0.0769	0.0182
2.0	0.2012	0.1929	0.1878	0.1854	0.1850	0.1837	0.1807	0.1771	0.1393	0.0749	0.0191

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                         GEO Tools
                          Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
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-------------------------------------------------------------
Stress coefficients according to GEDDES
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Stress coefficients for a point (end bearing) load,
Poisson's ratio = 0.20 [-]:

M/N	0.02	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.50	2.00
1.0	0.0970	0.0962	0.0937	0.0898	0.0847	0.0786	0.0615	0.0448	0.0208	0.0089
1.1	16.4294	3.7806	0.6196	0.2241	0.1334	0.1000	0.0660	0.0468	0.0223	0.0100
1.2	4.4545	2.7496	1.0019	0.3993	0.2059	0.1327	0.0725	0.0491	0.0236	0.0110
1.3	2.0433	1.6309	0.9246	0.4804	0.2676	0.1683	0.0812	0.0521	0.0250	0.0119
1.4	1.1809	1.0396	0.7340	0.4659	0.2930	0.1933	0.0906	0.0556	0.0263	0.0129
1.5	0.7765	0.7163	0.5690	0.4120	0.2879	0.2028	0.0986	0.0593	0.0277	0.0138
1.6	0.5543	0.5245	0.4464	0.3522	0.2668	0.1999	0.1040	0.0626	0.0291	0.0147
1.7	0.4187	0.4023	0.3574	0.2988	0.2402	0.1896	0.1063	0.0652	0.0304	0.0156
1.8	0.3295	0.3197	0.2922	0.2543	0.2136	0.1757	0.1059	0.0669	0.0316	0.0164
1.9	0.2674	0.2612	0.2434	0.2180	0.1892	0.1609	0.1034	0.0676	0.0326	0.0172
2.0	0.2223	0.2183	0.2062	0.1886	0.1678	0.1464	0.0996	0.0674	0.0334	0.0180

Stress coefficients for a point (end bearing) load,
Poisson's ratio = 0.30 [-]:

M/N	0.02	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.50	2.00
1.0	0.1023	0.1014	0.0988	0.0946	0.0890	0.0825	0.0642	0.0464	0.0210	0.0087
1.1	17.6966	3.9108	0.5987	0.2126	0.1289	0.0988	0.0669	0.0476	0.0223	0.0097
1.2	4.8034	2.9316	1.0373	0.4007	0.2030	0.1305	0.0723	0.0493	0.0235	0.0106
1.3	2.2027	1.7491	0.9770	0.4977	0.2721	0.1689	0.0809	0.0520	0.0247	0.0116
1.4	1.2722	1.1167	0.7816	0.4897	0.3036	0.1977	0.0910	0.0555	0.0260	0.0125
1.5	0.8359	0.7696	0.6078	0.4362	0.3016	0.2101	0.1000	0.0595	0.0274	0.0134
1.6	0.5962	0.5634	0.4774	0.3743	0.2813	0.2089	0.1064	0.0632	0.0288	0.0143
1.7	0.4498	0.4318	0.3824	0.3182	0.2542	0.1991	0.1096	0.0662	0.0302	0.0152
1.8	0.3536	0.3429	0.3126	0.2710	0.2265	0.1852	0.1098	0.0683	0.0315	0.0161
1.9	0.2867	0.2799	0.2603	0.2324	0.2009	0.1699	0.1077	0.0694	0.0327	0.0170
2.0	0.2381	0.2336	0.2204	0.2010	0.1783	0.1549	0.1041	0.0695	0.0337	0.0177

Stress coefficients for a point (end bearing) load,
Poisson's ratio = 0.40 [-]:

M/N	0.02	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.50	2.00
1.0	0.1094	0.1084	0.1055	0.1009	0.0949	0.0877	0.0677	0.0484	0.0212	0.0084
1.1	19.3862	4.0844	0.5707	0.1972	0.1230	0.0971	0.0681	0.0487	0.0223	0.0093
1.2	5.2686	3.1742	1.0844	0.4025	0.1991	0.1276	0.0721	0.0496	0.0233	0.0102
1.3	2.4152	1.9066	1.0469	0.5207	0.2780	0.1698	0.0805	0.0519	0.0244	0.0111
1.4	1.3940	1.2195	0.8450	0.5215	0.3178	0.2035	0.0914	0.0555	0.0257	0.0120
1.5	0.9151	0.8407	0.6596	0.4685	0.3198	0.2199	0.1018	0.0597	0.0270	0.0129
1.6	0.6519	0.6151	0.5188	0.4038	0.3005	0.2208	0.1096	0.0639	0.0285	0.0138
1.7	0.4913	0.4711	0.4157	0.3440	0.2728	0.2119	0.1139	0.0676	0.0300	0.0147
1.8	0.3858	0.3738	0.3398	0.2933	0.2437	0.1979	0.1150	0.0702	0.0315	0.0157
1.9	0.3124	0.3048	0.2829	0.2516	0.2164	0.1820	0.1134	0.0718	0.0328	0.0166
2.0	0.2591	0.2541	0.2393	0.2176	0.1922	0.1662	0.1100	0.0723	0.0340	0.0174

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                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
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Stress coefficients according to GEDDES
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Stress coefficients for uniform skin friction,
Poisson's ratio = 0.20 [-]:

M/N	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.50	1.00	2.00
1.0	6.4708	3.2375	2.1595	1.6202	1.2963	0.8630	0.6445	0.2300	0.0690	0.0081
1.1	1.7330	1.5946	1.4181	1.2418	1.0850	0.7953	0.6138	0.2283	0.0730	0.0096
1.2	0.8778	0.8577	0.8268	0.7881	0.7446	0.6318	0.5307	0.2231	0.0759	0.0111
1.3	0.5788	0.5728	0.5629	0.5498	0.5339	0.4867	0.4355	0.2138	0.0779	0.0125
1.4	0.4269	0.4243	0.4200	0.4141	0.4068	0.3838	0.3562	0.2010	0.0789	0.0139
1.5	0.3345	0.3335	0.3312	0.3281	0.3242	0.3114	0.2952	0.1862	0.0790	0.0152
1.6	0.2731	0.2722	0.2708	0.2690	0.2666	0.2588	0.2487	0.1708	0.0784	0.0165
1.7	0.2285	0.2281	0.2272	0.2261	0.2245	0.2195	0.2128	0.1559	0.0770	0.0175
1.8	0.1954	0.1950	0.1944	0.1936	0.1925	0.1890	0.1844	0.1420	0.0750	0.0185
1.9	0.1694	0.1692	0.1688	0.1682	0.1675	0.1650	0.1616	0.1293	0.0727	0.0193
2.0	0.1485	0.1487	0.1483	0.1479	0.1473	0.1455	0.1429	0.1180	0.0700	0.0201

Stress coefficients for uniform skin friction,
Poisson's ratio = 0.30 [-]:

M/N	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.50	1.00	2.00
1.0	6.8154	3.4044	2.2673	1.6984	1.3567	0.8998	0.6695	0.2346	0.0686	0.0076
1.1	1.8595	1.7074	1.5137	1.3210	1.1503	0.8368	0.6419	0.2335	0.0728	0.0091
1.2	0.9384	0.9164	0.8824	0.8399	0.7922	0.6688	0.5588	0.2292	0.0760	0.0105
1.3	0.6166	0.6101	0.5993	0.5848	0.5675	0.5157	0.4598	0.2207	0.0782	0.0120
1.4	0.4536	0.4507	0.4459	0.4395	0.4315	0.4062	0.3761	0.2082	0.0796	0.0134
1.5	0.3546	0.3533	0.3508	0.3474	0.3431	0.3291	0.3115	0.1934	0.0800	0.0148
1.6	0.2886	0.2877	0.2862	0.2842	0.2817	0.2731	0.2621	0.1777	0.0796	0.0160
1.7	0.2410	0.2406	0.2397	0.2384	0.2367	0.2312	0.2239	0.1623	0.0784	0.0172
1.8	0.2058	0.2053	0.2046	0.2038	0.2026	0.1988	0.1937	0.1479	0.0766	0.0182
1.9	0.1781	0.1778	0.1774	0.1768	0.1760	0.1732	0.1696	0.1347	0.0744	0.0191
2.0	0.1558	0.1560	0.1556	0.1552	0.1546	0.1526	0.1498	0.1229	0.0718	0.0199

Stress coefficients for uniform skin friction,
Poisson's ratio = 0.40 [-]:

M/N	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.50	1.00	2.00
1.0	7.2749	3.6271	2.4110	1.8026	1.4373	0.9489	0.7029	0.2407	0.0681	0.0069
1.1	2.0282	1.8576	1.6411	1.4266	1.2373	0.8921	0.6794	0.2404	0.0725	0.0083
1.2	1.0194	0.9946	0.9566	0.9089	0.8556	0.7182	0.5964	0.2373	0.0760	0.0098
1.3	0.6673	0.6599	0.6477	0.6316	0.6121	0.5544	0.4921	0.2298	0.0787	0.0113
1.4	0.4889	0.4859	0.4806	0.4734	0.4644	0.4362	0.4026	0.2178	0.0805	0.0128
1.5	0.3811	0.3797	0.3770	0.3731	0.3683	0.3527	0.3331	0.2029	0.0813	0.0142
1.6	0.3095	0.3084	0.3068	0.3045	0.3017	0.2922	0.2799	0.1868	0.0812	0.0155
1.7	0.2579	0.2573	0.2563	0.2548	0.2530	0.2469	0.2387	0.1708	0.0803	0.0167
1.8	0.2196	0.2190	0.2183	0.2174	0.2161	0.2119	0.2063	0.1557	0.0787	0.0178
1.9	0.1896	0.1894	0.1889	0.1882	0.1873	0.1843	0.1802	0.1419	0.0766	0.0188
2.0	0.1657	0.1658	0.1654	0.1649	0.1642	0.1620	0.1590	0.1294	0.0741	0.0196

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                         Version 10
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Stress coefficients according to GEDDES

Stress coefficients for linear variation of skin friction,
Poisson's ratio = 0.20 [-]:

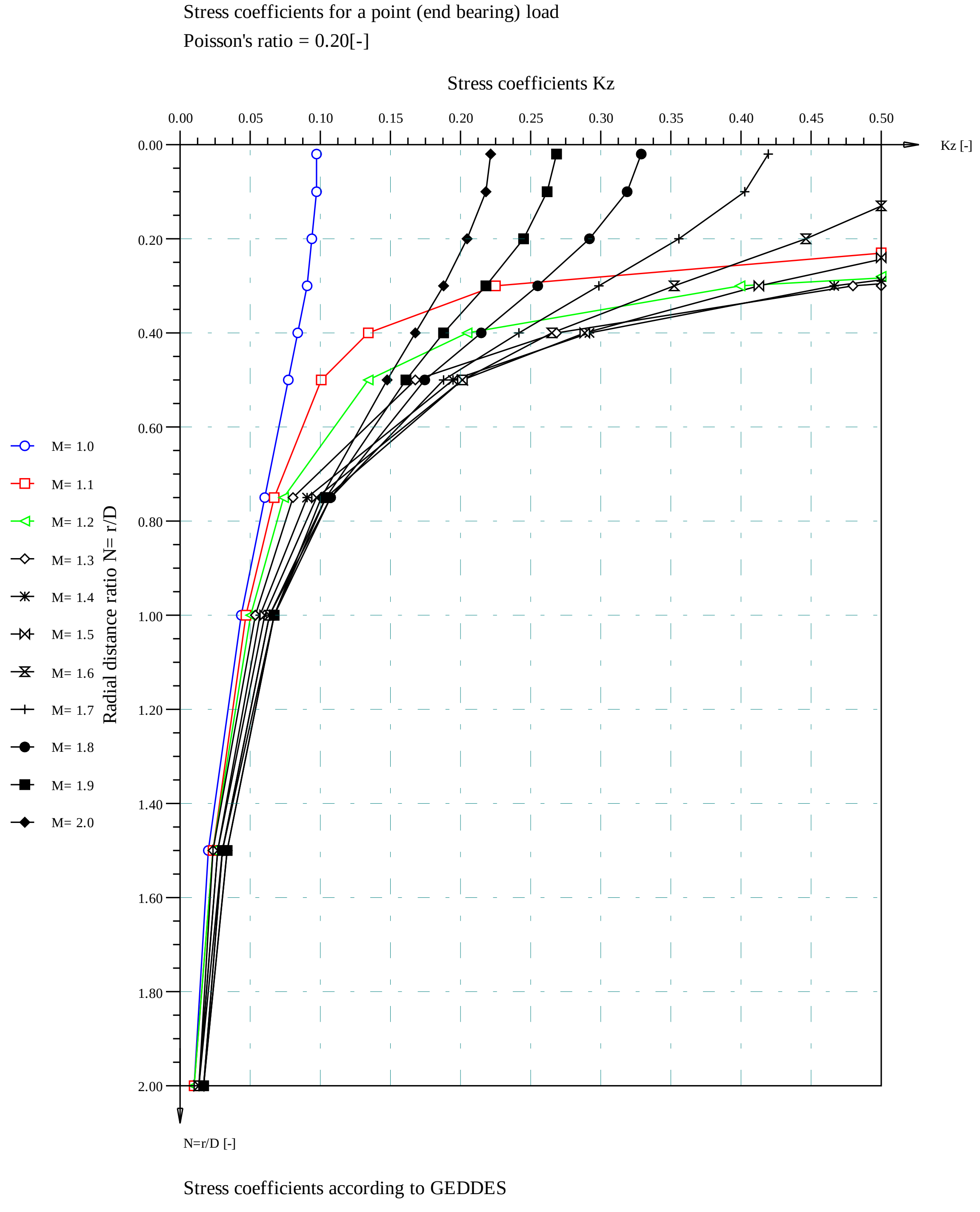
M/N	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.50	1.00	2.00
1.0	11.5296	5.3126	3.3017	2.3261	1.7582	1.0371	0.7033	0.1963	0.0618	0.0082
1.1	2.7516	2.4900	2.1590	1.8326	1.5466	1.0358	0.7345	0.2074	0.0656	0.0096
1.2	1.2526	1.2167	1.1616	1.0926	1.0160	0.8210	0.6528	0.2141	0.0689	0.0110
1.3	0.7695	0.7585	0.7416	0.7193	0.6926	0.6141	0.5311	0.2139	0.0717	0.0123
1.4	0.5391	0.5342	0.5273	0.5177	0.5060	0.4692	0.4261	0.2068	0.0738	0.0136
1.5	0.4078	0.4043	0.4007	0.3958	0.3898	0.3703	0.3460	0.1947	0.0750	0.0148
1.6	0.3214	0.3204	0.3184	0.3156	0.3122	0.3008	0.2861	0.1802	0.0754	0.0160
1.7	0.2631	0.2624	0.2613	0.2595	0.2575	0.2502	0.2408	0.1651	0.0750	0.0170
1.8	0.2203	0.2204	0.2196	0.2183	0.2169	0.2122	0.2058	0.1506	0.0739	0.0180
1.9	0.1888	0.1883	0.1878	0.1869	0.1860	0.1827	0.1782	0.1371	0.0722	0.0188
2.0	0.1651	0.1627	0.1630	0.1625	0.1618	0.1594	0.1561	0.1248	0.0700	0.0196

Stress coefficients for linear variation of skin friction,
Poisson's ratio = 0.30 [-]:

M/N	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.50	1.00	2.00
1.0	12.1288	5.5764	3.4585	2.4318	1.8346	1.0773	0.7276	0.1997	0.0616	0.0077
1.1	2.9619	2.6741	2.3110	1.9544	1.6430	1.0907	0.7679	0.2115	0.0654	0.0090
1.2	1.3462	1.3067	1.2461	1.1703	1.0863	0.8729	0.6899	0.2198	0.0689	0.0104
1.3	0.8258	0.8133	0.7946	0.7702	0.7408	0.6547	0.5639	0.2212	0.0720	0.0117
1.4	0.5770	0.5717	0.5641	0.5535	0.5407	0.5003	0.4530	0.2150	0.0744	0.0130
1.5	0.4352	0.4319	0.4279	0.4226	0.4160	0.3945	0.3679	0.2033	0.0760	0.0143
1.6	0.3430	0.3414	0.3394	0.3364	0.3326	0.3201	0.3040	0.1887	0.0768	0.0155
1.7	0.2806	0.2792	0.2781	0.2761	0.2739	0.2659	0.2556	0.1731	0.0767	0.0166
1.8	0.2340	0.2342	0.2333	0.2319	0.2304	0.2252	0.2182	0.1580	0.0758	0.0176
1.9	0.2004	0.1997	0.1993	0.1983	0.1972	0.1936	0.1887	0.1439	0.0742	0.0186
2.0	0.1753	0.1721	0.1726	0.1721	0.1714	0.1687	0.1651	0.1310	0.0721	0.0194

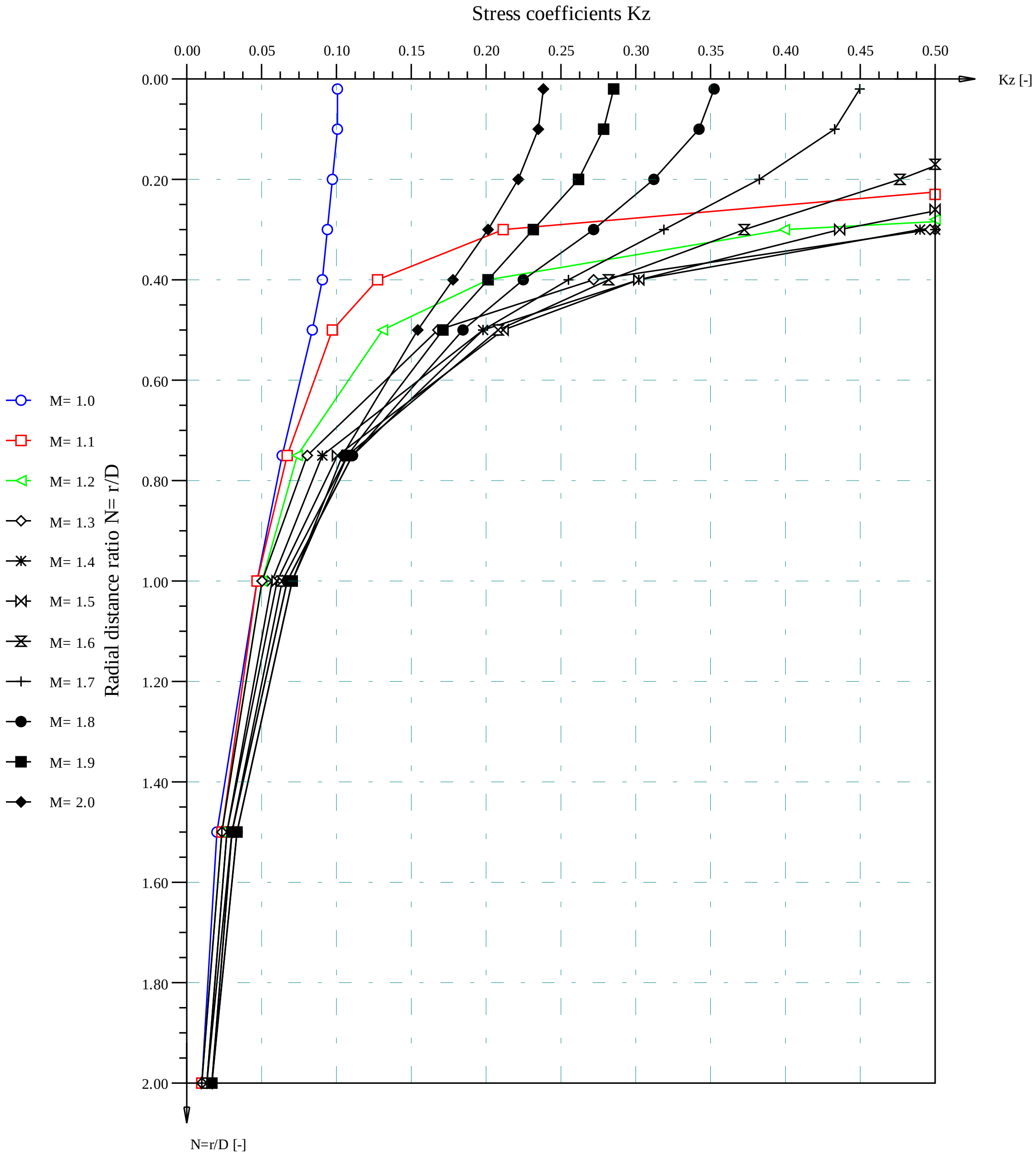
Stress coefficients for linear variation of skin friction,
Poisson's ratio = 0.40 [-]:

M/N	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.50	1.00	2.00
1.0	12.9279	5.9279	3.6675	2.5727	1.9365	1.1310	0.7600	0.2042	0.0614	0.0069
1.1	3.2425	2.9197	2.5138	2.1167	1.7716	1.1640	0.8125	0.2170	0.0652	0.0083
1.2	1.4712	1.4267	1.3587	1.2739	1.1799	0.9421	0.7393	0.2274	0.0689	0.0096
1.3	0.9001	0.8863	0.8655	0.8380	0.8051	0.7088	0.6076	0.2308	0.0723	0.0109
1.4	0.6281	0.6217	0.6132	0.6014	0.5870	0.5417	0.4890	0.2259	0.0752	0.0123
1.5	0.4729	0.4687	0.4642	0.4582	0.4508	0.4268	0.3971	0.2147	0.0773	0.0136
1.6	0.3707	0.3698	0.3674	0.3640	0.3598	0.3459	0.3279	0.1999	0.0786	0.0149
1.7	0.3040	0.3019	0.3005	0.2983	0.2957	0.2869	0.2753	0.1838	0.0788	0.0161
1.8	0.2523	0.2525	0.2516	0.2500	0.2483	0.2425	0.2347	0.1679	0.0782	0.0172
1.9	0.2160	0.2149	0.2144	0.2134	0.2122	0.2082	0.2027	0.1530	0.0769	0.0182
2.0	0.1890	0.1853	0.1854	0.1849	0.1841	0.1811	0.1771	0.1393	0.0749	0.0191



GEOTEC Software Inc., Calgary AB, Canada T3E 7Y7 PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323	
Scale: 3	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.2
File: Point-Poisson ratio 0.2	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Point load)

Stress coefficients for a point (end bearing) load
Poisson's ratio = 0.30[-]



Stress coefficients according to GEDDES

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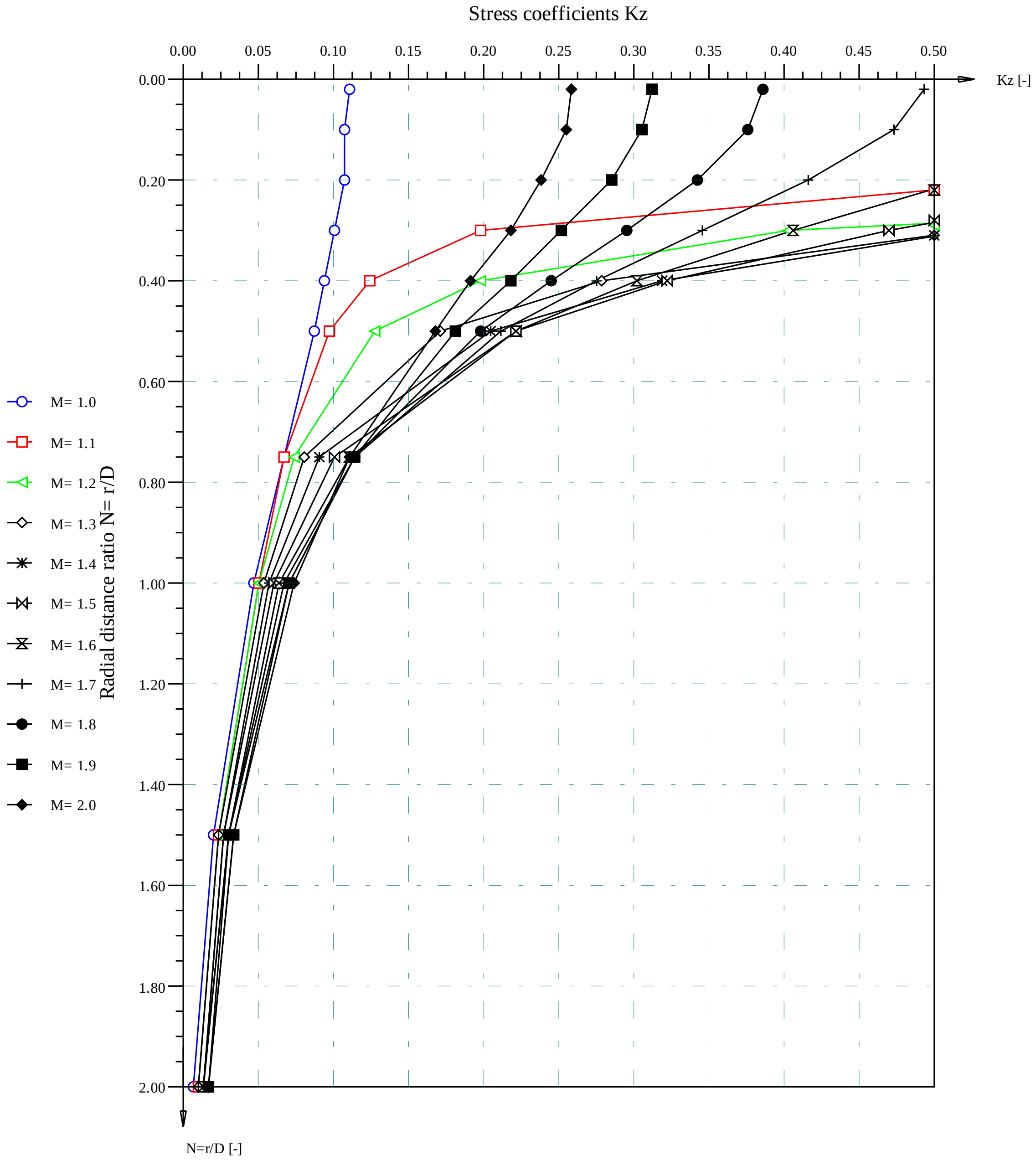
Scale: 3	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.3
File: Point-Poisson ratio 0.3	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Point load)

Stress coefficients for a point (end bearing) load
Poisson's ratio = 0.40[-]



Stress coefficients according to GEDDES

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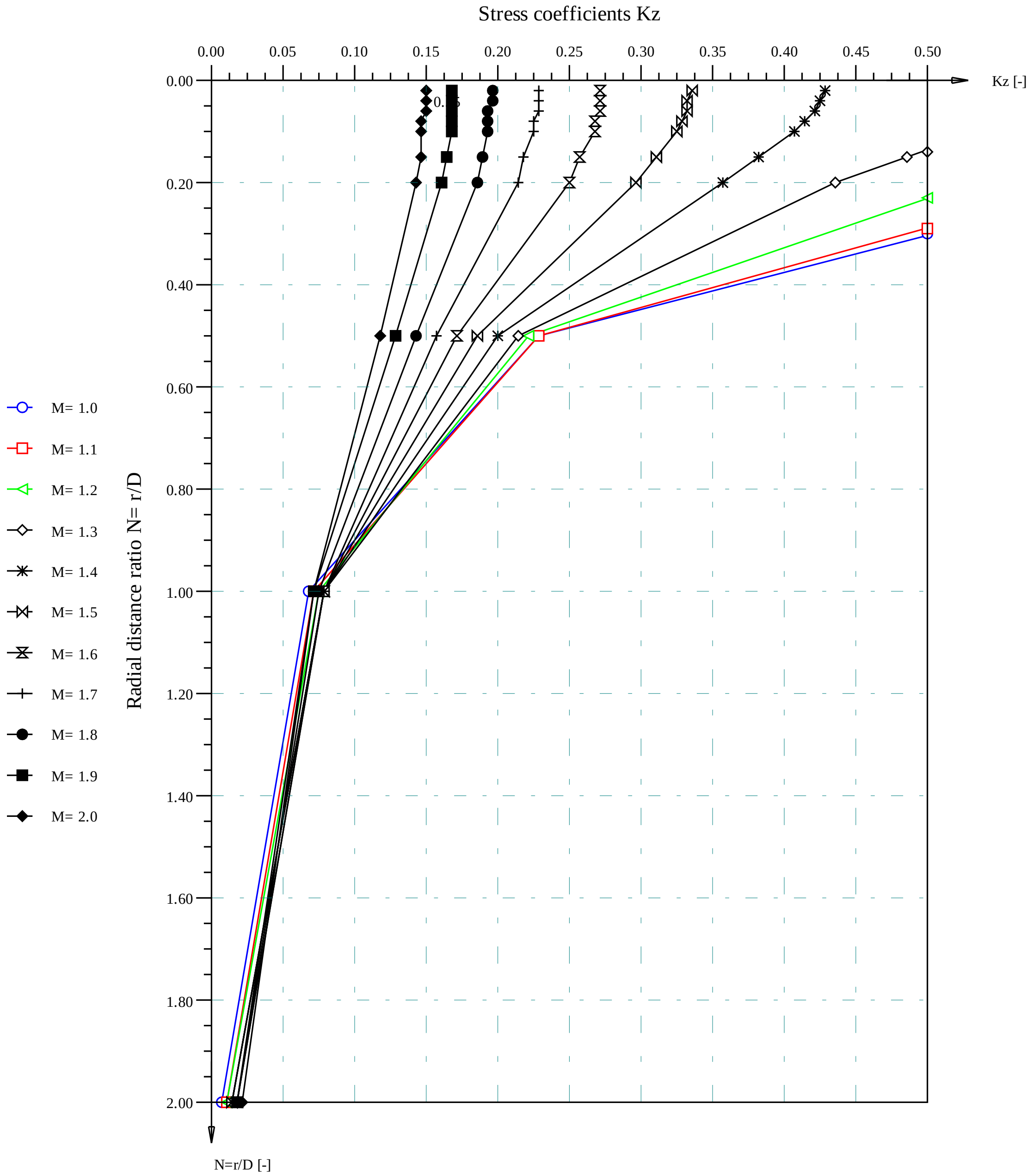
Scale: 3	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.4
File: Point-Poisson ratio 0.4	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Point load)

Stress coefficients for uniform skin friction

Poisson's ratio = 0.20[-]



Stress coefficients according to GEDDES

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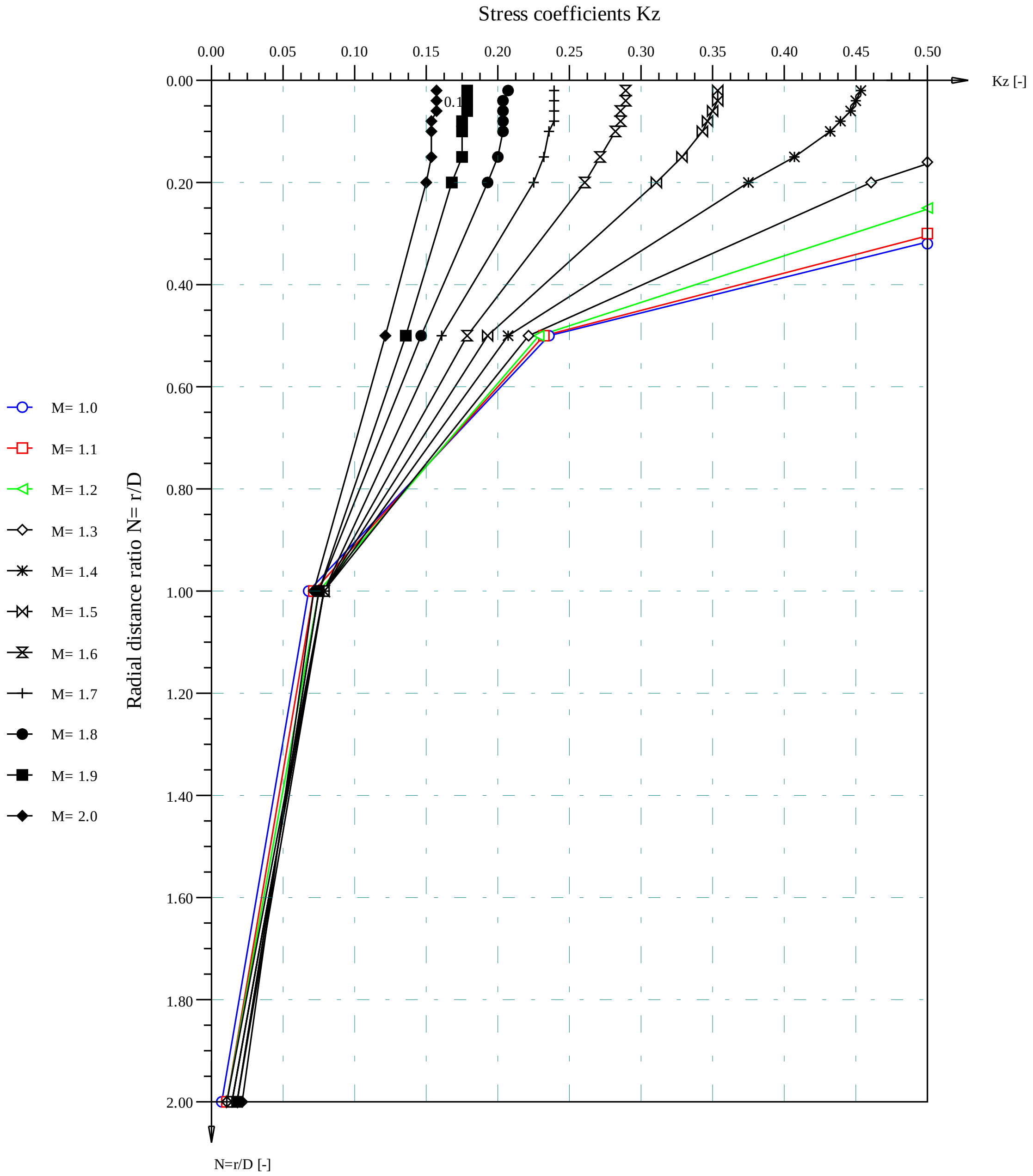
Scale: 4	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.2
File: Uniform-Poisson ratio 0.2	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Uniform skin friction)

Stress coefficients for uniform skin friction

Poisson's ratio = 0.30[-]

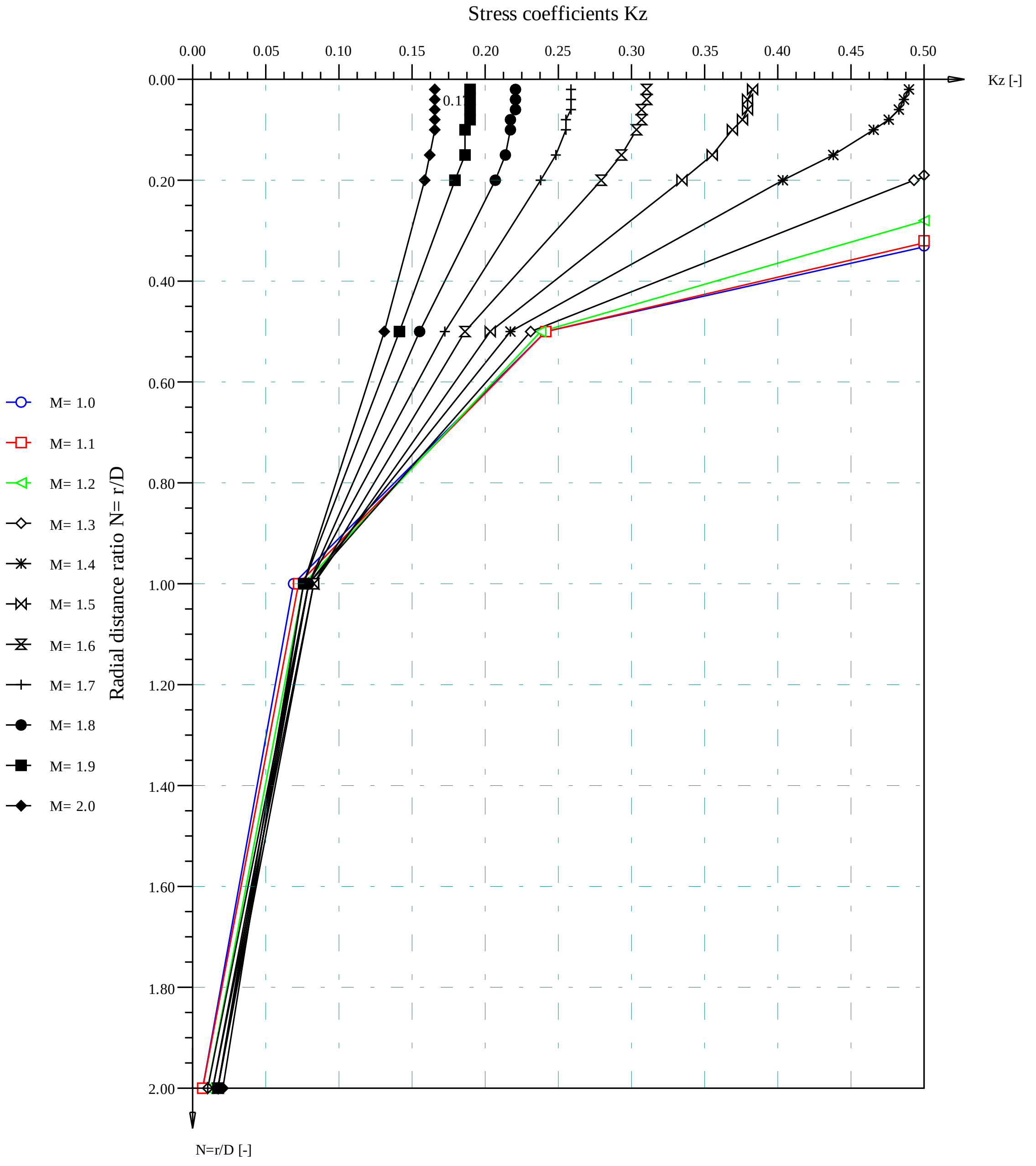


Stress coefficients according to GEDDES

GEOTEC Software Inc., Calgary AB, Canada T3E 7Y7 PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323	
Scale: 4	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.3
File: Uniform-Poisson ratio 0.3	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Uniform skin friction)

Stress coefficients for uniform skin friction

Poisson's ratio = 0.40[-]



Stress coefficients according to GEDDES

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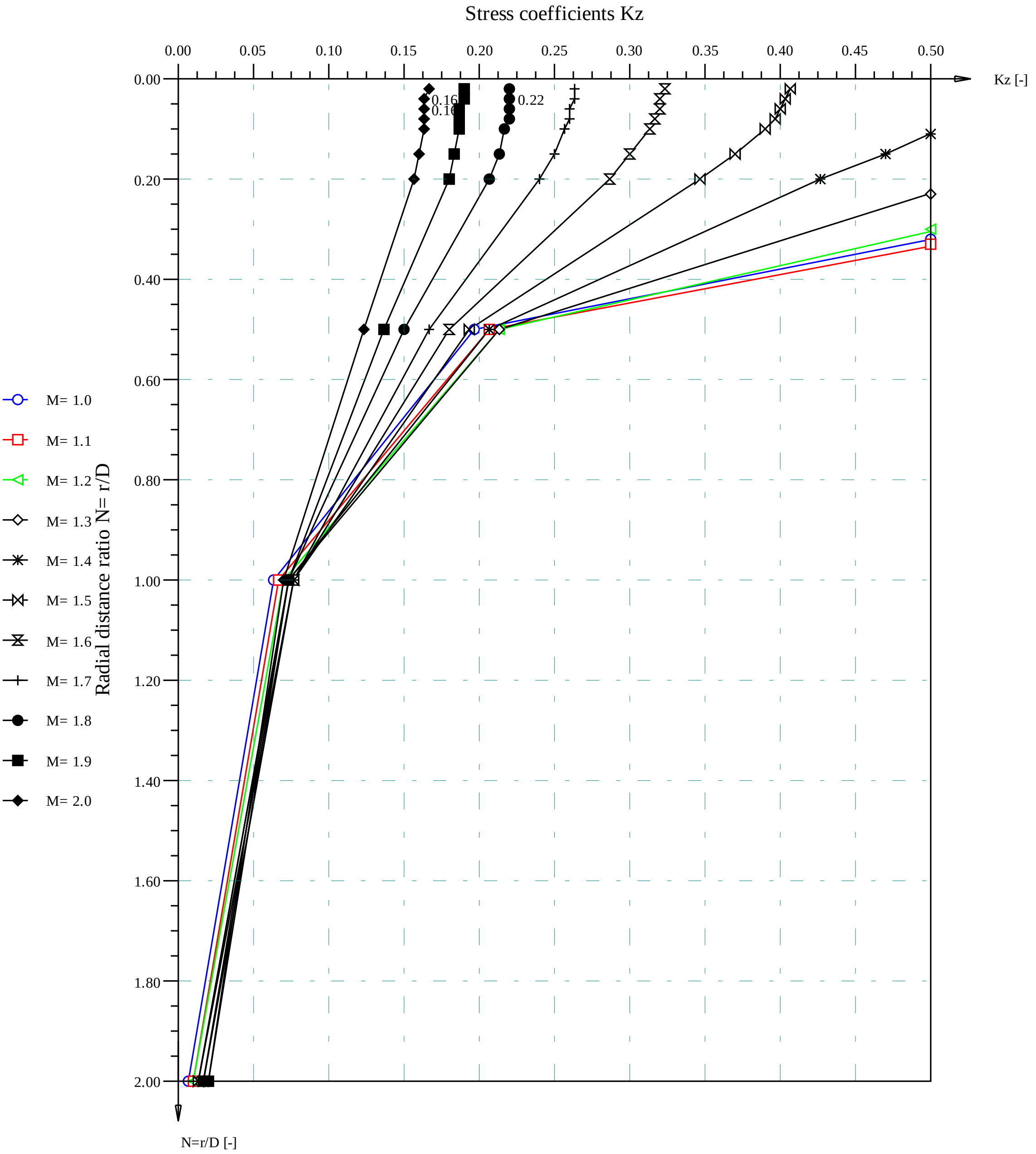
PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323

Scale: 3	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.4
File: Uniform-Poisson ratio 0.4	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Uniform skin friction)

Stress coefficients for linear variation of skin friction

Poisson's ratio = 0.20[-]



Stress coefficients according to GEDDES

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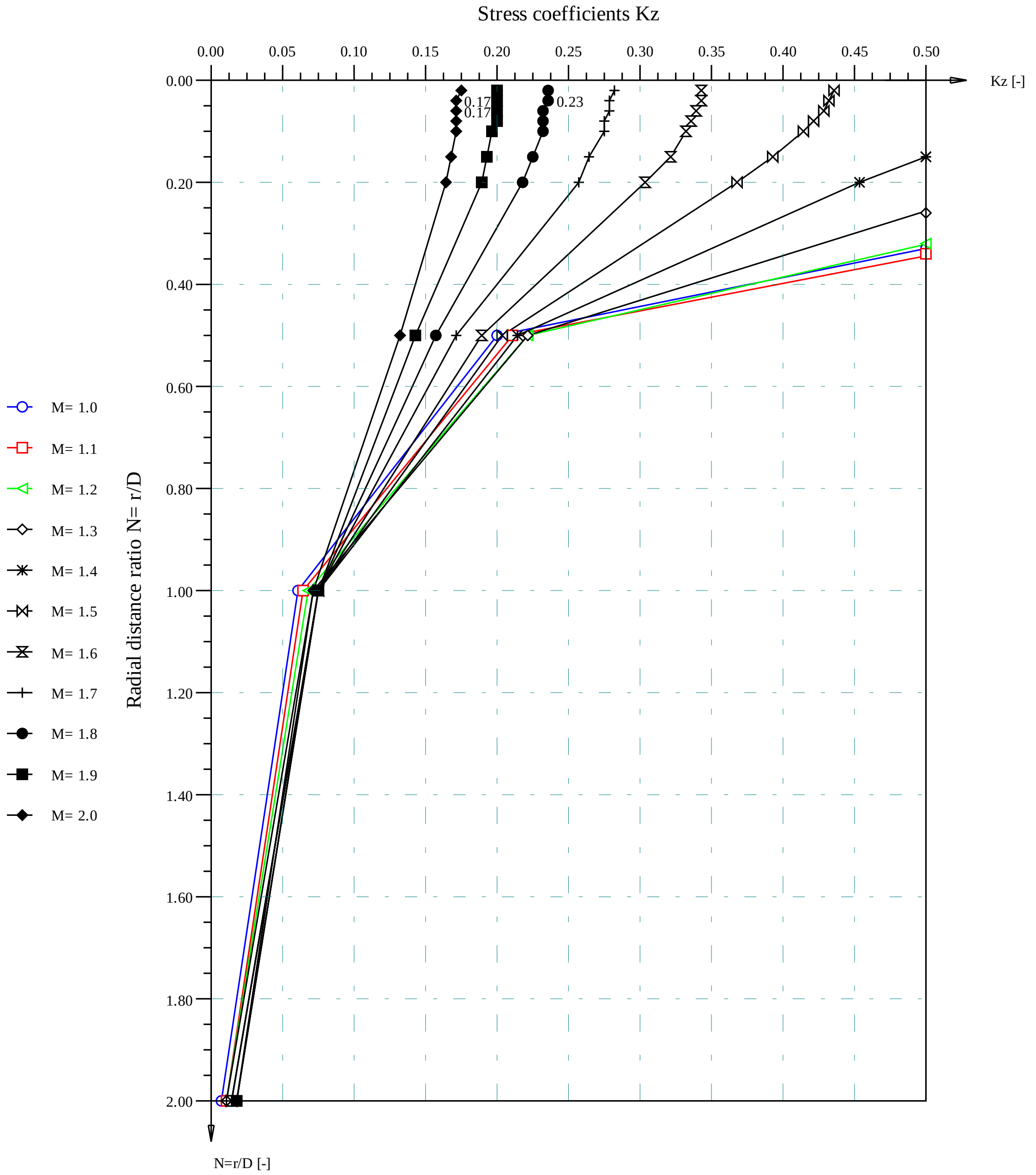
Scale: 3	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.2
File: Linear-Poisson ratio 0.2	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Linear variation of skin friction)

Stress coefficients for linear variation of skin friction

Poisson's ratio = 0.30[-]



Stress coefficients according to GEDDES

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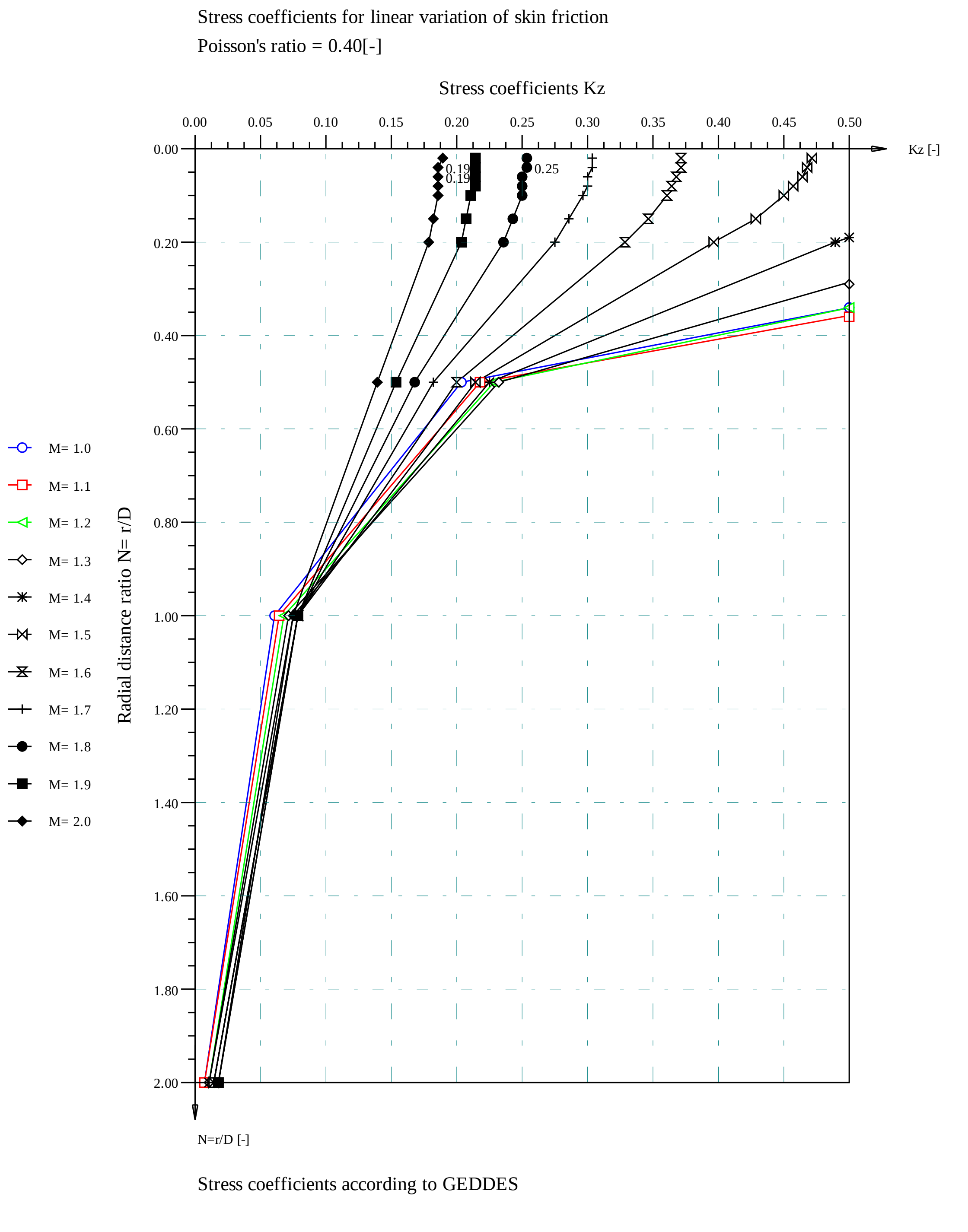
Scale: 4	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.3
File: Linear-Poisson ratio 0.3	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Linear variation of skin friction)



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Scale: 4	Project: Foundation Analysis and Design, Bowels.Poisson ratio=0.3
File: Linear-Poisson ratio 0.4	Date: 11_06_2015
Page No.:	Title: Stress coefficients according to GEDDES (Linear variation of skin friction)

4.7 Examples to estimate settlement of pile caps using stress coefficients

Hand calculations of settlement in soil are usually performed using tabulated values of the influence factor. Therefore, the stress coefficients according to *GEDDES* is used to obtain a vertical stress in soil. Consequently, settlement in soil below the pile tips of an pile cap is possible manually. The settlement is estimated beneath the center of the free standing pile cap, which is the zone of most interest in settlement analyses.

In all examples to estimate settlement of free standing pile caps using stress coefficients, piles are end bearing piles and the soil is assumed to have *Poisson*'s ratio of ν_s = 0.3 [-], as *Poisson*'s ratio in general ranges between 0 and 0.5. Stress coefficients according to *GEDDES* from Eqns 4.3 to 4.5 when *Poisso*'s ratio ν_s = 0.3 [-] for end bearing piles are tabulated in Table 4.6.

4.7.1 Example 3: Settlement at the center of a circular pile cap

For the shown piles in Figure 4.10, determine the settlement for a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] below the pile tips. Piles have a length of l = 17 [m]. Modulus of Compressibility of the clay is E_s =1000 [kN/m^2]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume uniform friction piles.

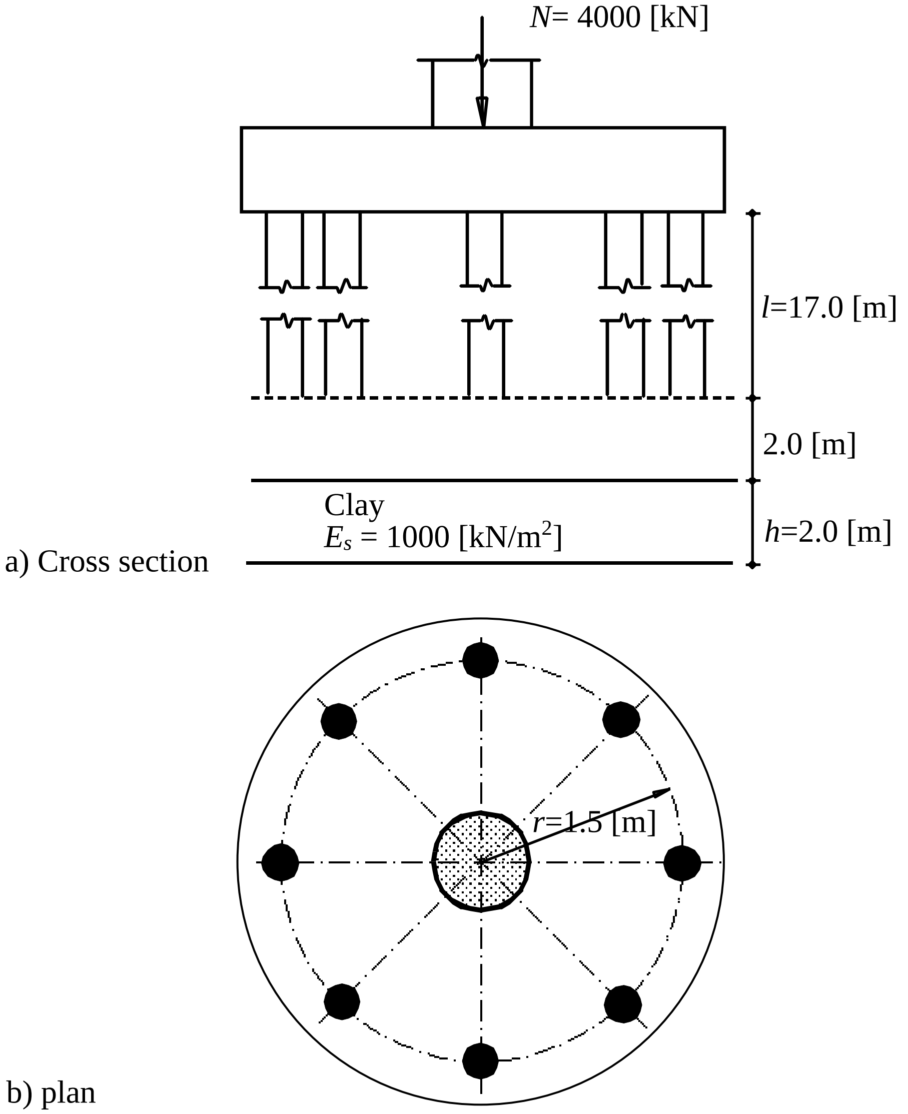

Figure 4.10 Pile cap with piles and the soil

Force in a single pile:

$$P_p = \frac{4000}{8} = 500 \text{ [kN]}$$

Depth and radial distance ratios:

$$M = \frac{z}{l} = \frac{20}{17} = 1.176 \text{ [-]} \approx 1.2 \text{ [-]}$$

$$N = \frac{r}{l} = \frac{1.5}{17} = 0.088 \text{ [-]} \approx 0.1 \text{ [-]}$$

From Table 4.6 the vertical stress coefficient is:

$$K_z = 0.7922 \text{ [-]}$$

Stress in the clay layer:

$$\sigma_z = \frac{p}{l^2} K_z = \frac{500}{17^2} 0.7922 = 1.37 \text{ [kN/m}^2\text{]}$$

$$\sigma_{z(\text{total})} = 8 \times 1.37 = 10.96 \text{ [kN/m}^2\text{]}$$

Settlement in the soil layer:

$$s = \frac{1}{E_s} \sigma_{z(\text{total})} h = \frac{1}{1000} \times 10.96 \times 2 = 0.022 \text{ [m]}$$

$$s = 2.2 \text{ [cm]}$$

4.7.2 Example 4: Settlement at the center of a square pile cap

Determine the settlement under the center of the square pile cape shown in Figure 4.11. The pile cap is loaded by a central vertical load of N = 4000 [kN]. The settlement is required for a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] below the pile tips as shown in Figure 4.11. Piles have a length of l = 10 [m]. The Modulus of Compressibility of the clay E_s = 2000 [kN/m^2]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume uniform friction piles.

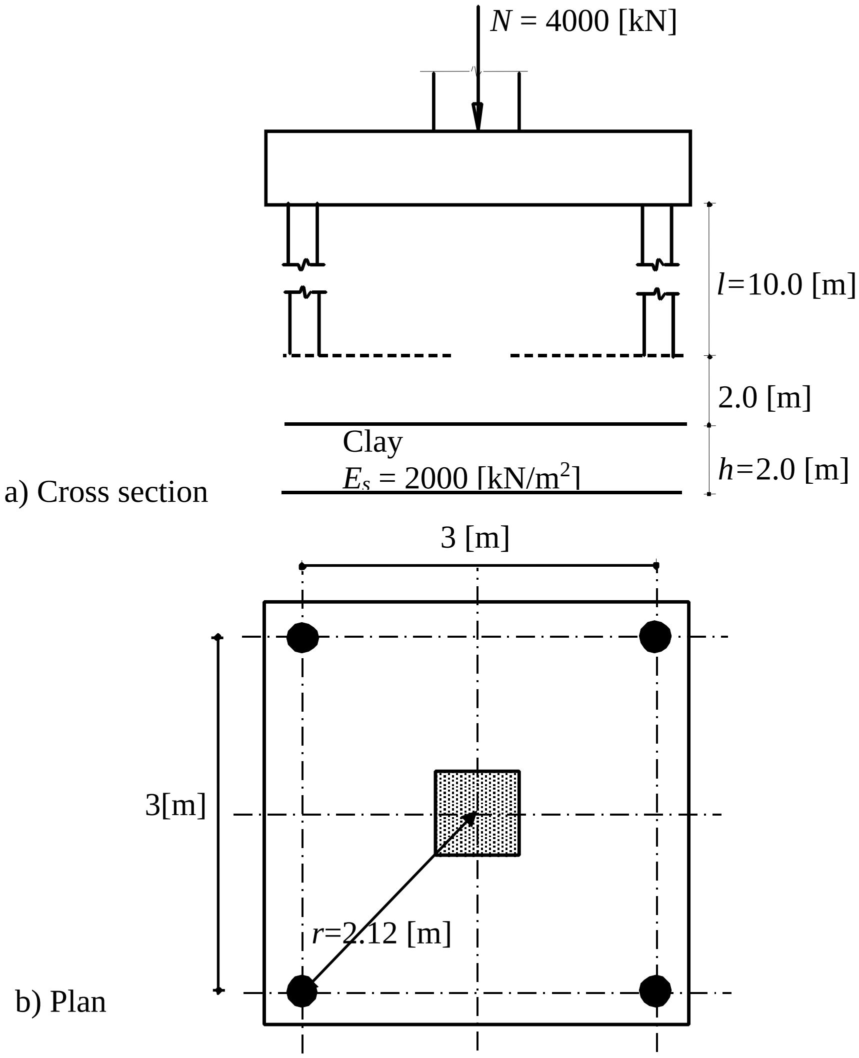

Figure 4.11 Pile cap with piles and the soil

Force in a single pile:

$$P_p = \frac{4000}{4} = 1000 \text{ [kN]}$$

Depth and radial distance ratios:

$$M = \frac{z}{l} = \frac{13}{10} = 1.3 \ [\text{-}]$$

$$N = \frac{r}{l} = \frac{\sqrt{1.5^2 + 1.5^2}}{10} = \frac{2.12}{10} = 0.2 \ [\text{-}]$$

From Table 4.6 the vertical stress coefficient is:

$$K_z = 0.4598 \ [\text{-}]$$

Stress in the clay layer:

$$\sigma_z = \frac{p}{l^2} K_z = \frac{1000}{10^2} 0.4598 = 4.6 \ [\text{kN/m}^2]$$

$$\sigma_{z(\text{total})} = 4 \times 4.6 = 18.4 \ [\text{kN/m}^2]$$

Settlement in the soil layer:

$$s = \frac{1}{E_s} \sigma_{z(\text{total})} h = \frac{1}{2000} \times 18.4 \times 2 = 0.0184 \ [\text{m}]$$

$$s = 1.84 \ [\text{cm}]$$

4.7.3 Example 5: Settlement at the center of a rectangular pile cap

An estimation of the settlement under the center of the pile cape in Figure 4.12 is to be carried out. The settlement is required for a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] from the pile tips as shown in Figure 4.12. Piles have a length of l = 15 [m]. Modulus of compressibility of the clay is E_s=8000 [kN/m^2]. *Poission's* ratio of the soil is ν_s=0.3 [-]. Assume uniform friction piles where the stress coefficients are listed in Table 4.6.

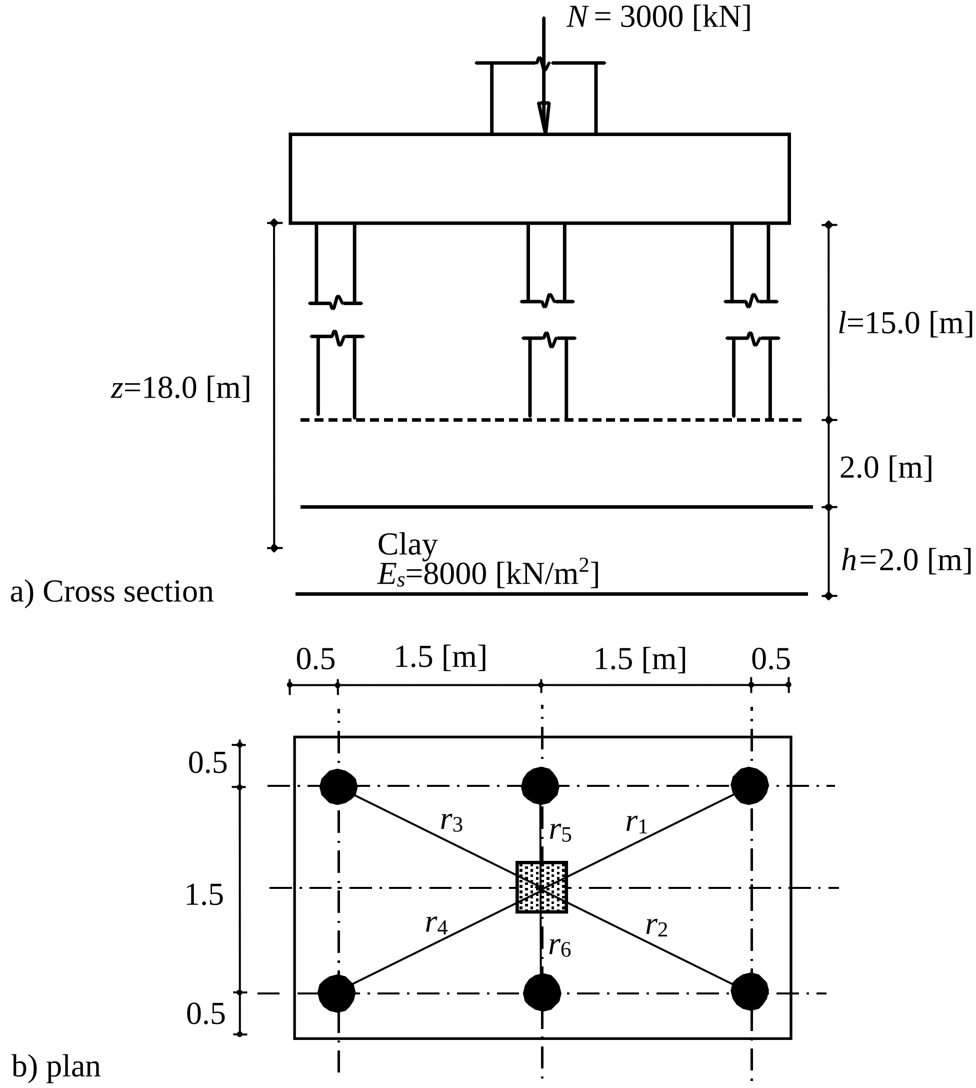

Figure 4.12 Pile cap with piles and the soil

Determining distance r:

For a corner pile $r_1 = r_2 = r_3 = r_3 = \sqrt{1.5^2 + 0.75^2} = 1.67$ [m]

For an edge pile $r_5 = r_6 = 0.75$ [m]

Determining dimensionless factors N, M:

$$\text{For a corner pile } N_1 = \frac{r_1}{l} = \frac{1.67}{15} = 0.11 \text{ [-]}$$

$$\text{For an edge pile } N_2 = \frac{r_5}{l} = \frac{0.75}{15} = 0.05 \text{ [-]}$$

$$M_1 = M_2 = \frac{z}{l} = \frac{18}{15} = 1.2 \text{ [-]}$$

From Table 4.6, the vertical stress coefficients are:

$$\text{For a corner pile } K_{z1} = 0.7689 \text{ [-]}$$

$$\text{For an edge pile } K_{z2} = 0.8994 \text{ [-]}$$

Load of a single pile:

$$P = \frac{N}{n_p} = \frac{3000}{6} = 500 \text{ [kN]}$$

Stress in the layer due to a single pile load:

$$\text{Due to a corner pile } \sigma_{z1} = \frac{P}{l^2} K_{z1} = \frac{500}{15^2} \times 0.7689 = 1.7 \text{ [kN/m}^2\text{]}$$

$$\text{Due to an edge pile } \sigma_{z2} = \frac{P}{l^2} K_{z2} = \frac{500}{15^2} \times 0.8994 = 2.0 \text{ [kN/m}^2\text{]}$$

Settlement in the layer due to a single pile load:

$$\text{Due to a corner pile } s_{p1} = \frac{1}{Es} \sigma_{z1} h = \frac{1}{8000} \times 1.7 \times 2 = 4.25 \times 10^{-4} \text{ [m]} = 0.425 \text{ [mm]}$$

$$\text{Due to an edge pile } s_{p2} = \frac{1}{Es} \sigma_{z2} h = \frac{1}{8000} \times 2.0 \times 2 = 5 \times 10^{-4} \text{ [m]} = 0.5 \text{ [mm]}$$

Total settlement in the layer at the center of the pile cap:

$$s_t = 4 s_{p1} + 2 s_{p2} = 4 \times 0.425 + 2 \times 0.5 = 2.7 \text{ [mm]}$$

4.7.4 Example 6: Settlement at the center of a square pile cap

An estimation of the settlement under the center of the pile cape carries a load of N=4000 [kN] in Figure 4.13 is to be carried out. The settlement is required for a clay layer of 2 [m] thickness and lies at a depth of 1 [m] below the pile tips. Piles have a length of l = 12 [m]. Modulus of compressibility of the clay is E_s=8000 [kN/m^2]. *Poission's* ratio of the soil is ν_s=0.3 [-]. Assume end bearing piles, where the stress coefficients are listed in Table 4.6.

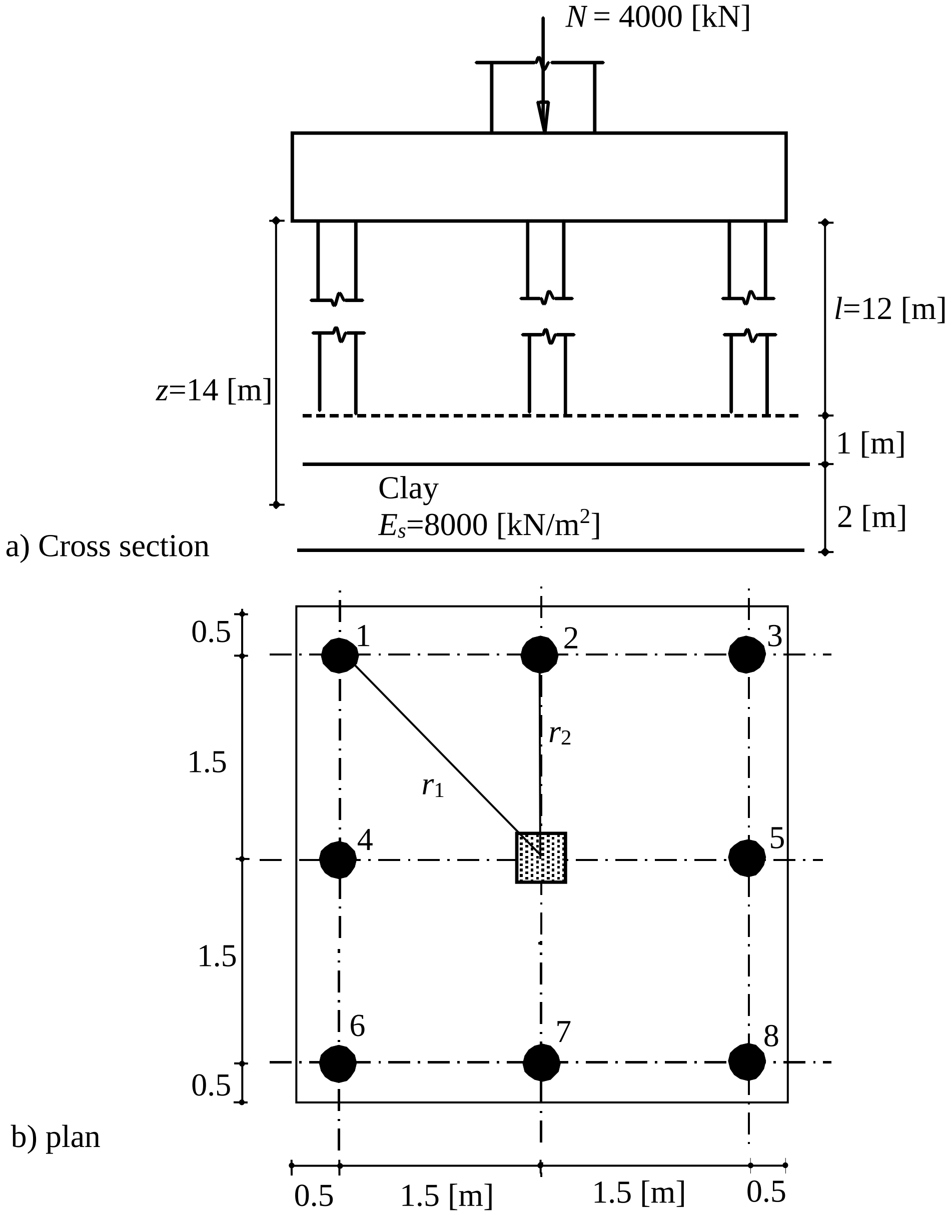

Figure 4.13 Pile cap with piles and the soil

Determining distance *r*:

$$\text{For a corner pile } r_1 = r_3 = r_6 = r_8 = \sqrt{1.5^2 + 1.5^2} = 2.12 \text{ [m]}$$
$$\text{For an edge pile } r_2 = r_4 = r_5 = r_7 = 1.5 \text{ [m]}$$

Determining dimensionless factors *N*, *M*:

$$\text{For a corner pile } N_1 = \frac{r_1}{l} = \frac{2.12}{12} = 0.18 \approx 0.2 \text{ [-]}$$
$$\text{For an edge pile } N_2 = \frac{r_2}{l} = \frac{1.5}{12} = 0.125 \approx 0.1 \text{ [-]}$$
$$M_1 = M_2 = \frac{z}{l} = \frac{14}{12} = 1.17 \approx 1.2 \text{ [-]}$$

From Table 4.6, the vertical stress coefficients are:

$$\text{For a corner pile } K_{z1} = 1.0373 \text{ [-]}$$
$$\text{For an edge pile } K_{z2} = 2.9316 \text{ [-]}$$

Load of a single pile:

$$P = \frac{N}{n_p} = \frac{4000}{6} = 500 \text{ [kN]}$$

Stress in the layer due to a single pile load:

$$\text{Due to a corner pile } \sigma_{z1} = \frac{P}{l^2} K_{z1} = \frac{500}{12^2} \times 1.0373 = 3.6 \text{ [kN/m}^2\text{]}$$
$$\text{Due to an edge pile } \sigma_{z2} = \frac{P}{l^2} K_{z2} = \frac{500}{12^2} \times 2.9316 = 10.18 \text{ [kN/m}^2\text{]}$$

Settlement in the layer due to a single pile load:

$$\text{Due to a corner pile } s_{p1} = \frac{1}{E_s} \sigma_{z1} h = \frac{1}{8000} \times 3.6 \times 2 = 9 \times 10^{-4} \text{ [m]} = 0.09 \text{ [cm]}$$
$$\text{Due to an edge pile } s_{p2} = \frac{1}{E_s} \sigma_{z2} h = \frac{1}{8000} \times 10.18 \times 2 = 25.45 \times 10^{-4} \text{ [m]} = 0.25 \text{ [cm]}$$

Total settlement in the layer at the center of the pile cap:

$$s_t = 4 s_{p1} + 4 s_{p2} = 4 \times (0.09 + 0.25) = 1.38 \text{ [cm]}$$

4.8 Examples to estimate settlement of pile groups using stress coefficients

Hand calculations of pile group settlement under a rigid or a flexible free standing pile cap is also possible by using stress coefficients according to *GEDDES*. Rigid symmetrical free standing pile cap under eccentric applied load will settle uniformly downward. It means, settlement in all piles will be the same. Therefore, the unknowns of the problem are n pile reactions P_i and the rigid body translation s_o. The derivation of the uniform settlement for the rigid free standing pile cap can be carried out by equating the settlement s_i by a uniform translation s_o at all piles.

In all examples to estimate pile group settlement using stress coefficients, piles are friction piles and the soil is assumed to have *Poisson*'s ratio of ν_s = 0.3 [-], as *Poisson*'s ratio in general ranges between 0 and 0.5. Stress coefficients according to *GEDDES* from Eqns 4.3 to 4.5 when *Poisso*'s ratio ν_s = 0.3 [-] for friction piles are tabulated in Table 4.6.

4.8.1 Example 7: Settlement of pile group under a rectangular flexible pile cap

Figure 4.14 shows a group of piles under a square free standing flexible pile cap. Piles have a length of l = 10 [m], a diameter of φ=0.3 [m] and a Modulus of Elasticity of E_p=3×10^7 [kN/m^2]. The pile cap is loaded by a central vertical load of N = 6000 [kN]. Determine the pile group settlement due to a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] below the pile tips. The Modulus of Compressibility of the clay E_s=5000 [kN/m^2]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume uniform friction piles and suppose free standing flexible pile cap.

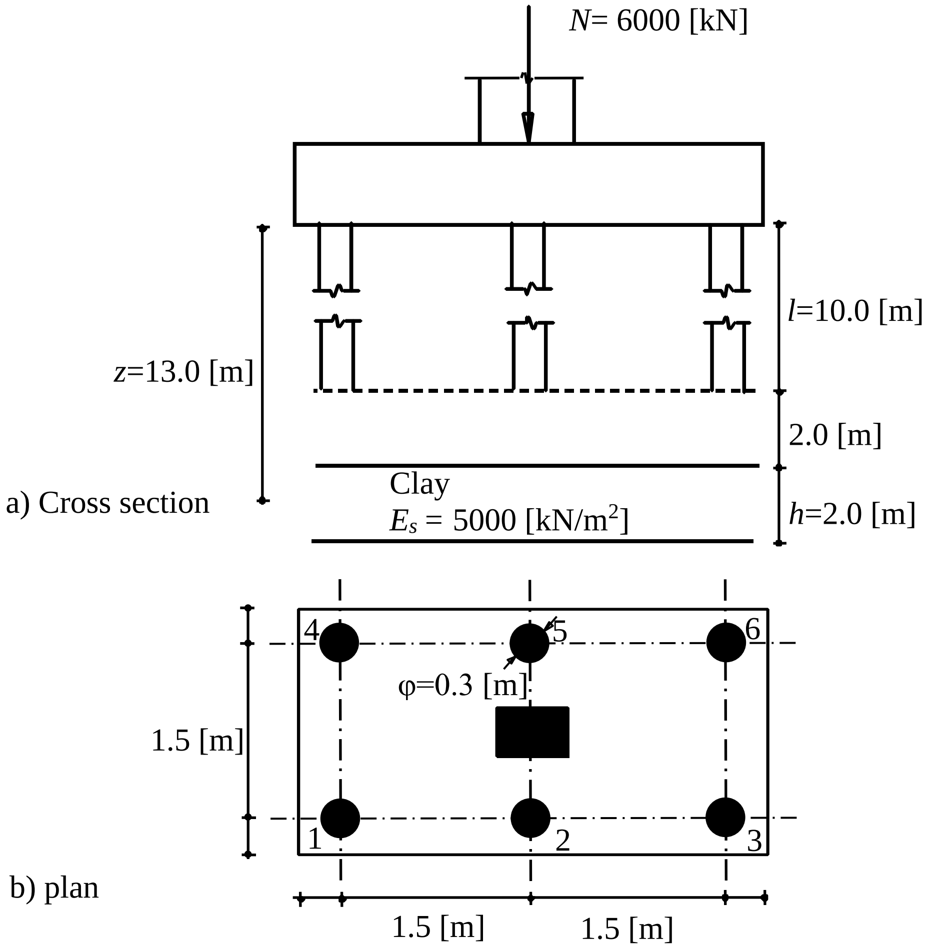

Figure 4.14 Pile cap with piles and the soil

For free standing flexible pile cap:

$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = 500 \text{ [kN]}$$

Due to symmetry of pile group in geometry and load:

$$s_1 = s_3 = s_4 = s_6$$
$$s_2 = s_5$$

Elastic shortening of the pile:

$$\Delta = \frac{P\,l}{A_p E_p} = \frac{500 \times 10}{\frac{\pi 0.3^2}{4} 3 \times 10^7}$$
$$\Delta = 2.36 \times 10^{-3} \text{ [m]}$$

Depth ratio is:

$$M = \frac{z}{l} = \frac{13}{10} = 1.3 \text{ [-]}$$

Determining radial distance r_{1j} between corner pile 1 and other piles (Figure 4.15):

$$r_{11} = 0$$
$$r_{12} = r_{14} = 1.5 \text{ [m]}$$
$$r_{13} = 3 \text{ [m]}$$
$$r_{15} = \sqrt{1.5^2 + 1.5^2} = 2.12 \text{ [m]}$$
$$r_{16} = \sqrt{3^2 + 1.5^2} = 3.35 \text{ [m]}$$

Radial distance ratios for pile 1:

$$N_{11} = 0$$
$$N_{12} = N_{14} = \frac{r_{12}}{l} = \frac{1.5}{10} = 0.15 \text{ [-]}$$
$$N_{13} = \frac{r_{13}}{l} = \frac{3}{10} = 0.3 \text{ [-]}$$
$$N_{15} = \frac{r_{15}}{l} = \frac{2.12}{10} = 0.21 \text{ [-]}$$
$$N_{16} = \frac{r_{16}}{l} = \frac{3.35}{10} = 0.34 \text{ [-]}$$

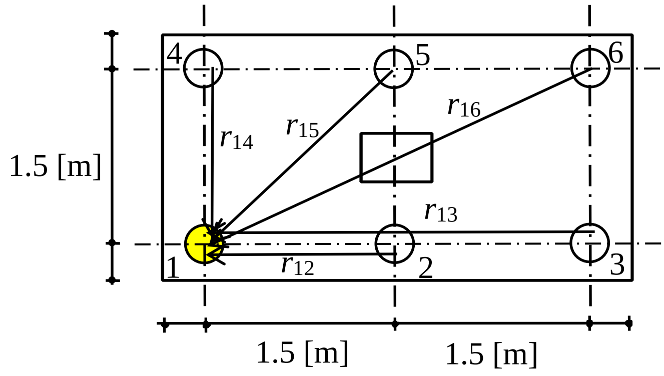

Figure 4.15 Radial distance r_{1j} between corner pile 1 and other piles

From Table 4.6, vertical stress coefficients K_{z1j} for pile 1 due to pile j are:

$$K_{z11} = 0.6335 \text{ [-]}$$

$$K_{z12} = K_{z14} = 0.5157 \text{ [-]}$$

$$K_{z13} = 0.3577 \text{ [-]}$$

$$K_{z15} = 0.4487 \text{ [-]}$$

$$K_{z16} = 0.3234 \text{ [-]}$$

Stress in the clay layer at pile 1:

$$\sigma_{z1} = \frac{P_1}{l^2}K_{z11} + \frac{P_2}{l^2}K_{z12} + \frac{P_3}{l^2}K_{z13} + \frac{P_4}{l^2}K_{z14} + \frac{P_5}{l^2}K_{z15} + \frac{P_6}{l^2}K_{z16}$$

$$\sigma_{z1} = \frac{500}{10^2}(0.6335 + 0.5157 + 0.3577 + 0.5157 + 0.4487 + 0.3234)$$

$$\sigma_{z1} = 13.97 \text{ [kN/m}^2\text{]}$$

Displacement of pile 1:

$$s_1 = \frac{1}{E_s}\sigma_{z1}h = \frac{13.97 \times 2}{5000}$$

$$s_1 = 5.88 \times 10^{-3} \text{ [m]}$$

Settlement of pile 1:

$$s_{P1} = s_1 + \Delta$$

$$s_{P1} = 5.88 \times 10^{-3} + 2.36 \times 10^{-3}$$

$$s_{P1} = 8.24 \times 10^{-3} \text{ [m]} = 8.24 \text{ [mm]}$$

Determining radial distance r_{2j} between edge pile 2 and other piles (Figure 4.16):

$$r_{21} = r_{23} = r_{25} = 1.5 \text{ [m]}$$
$$r_{22} = 0$$
$$r_{24} = r_{26} = \sqrt{1.5^2 + 1.5^2} = 2.12 \text{ [m]}$$

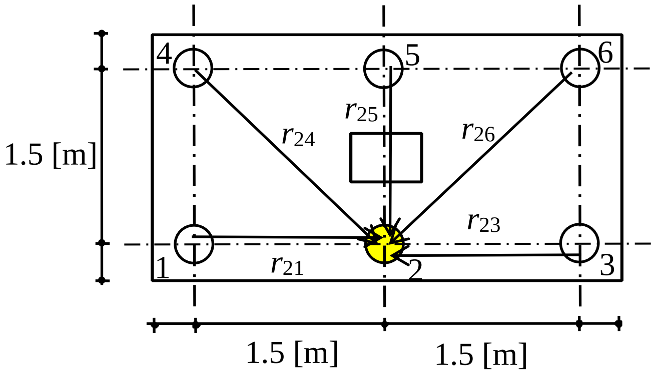

Figure 4.16 Radial distance r_{2j} between edge pile 2 and other piles

Radial distance ratios for pile 2:

$$N_{21} = N_{23} = N_{25} = \frac{r_{21}}{l} = \frac{1.5}{10} = 0.15 \text{ [-]}$$
$$N_{22} = 0 \text{ [-]}$$
$$N_{24} = N_{26} = \frac{r_{24}}{l} = \frac{2.12}{10} = 0.21 \text{ [-]}$$

From Table 4.6, vertical stress coefficients K_{z2j} for pile 2 due to pile j are:

$$K_{z21} = K_{z23} = K_{z25} = 0.5157 \text{ [-]}$$

$$K_{z22} = 0.6335 \text{ [-]}$$

$$K_{z24} = K_{z26} = 0.4487 \text{ [-]}$$

Stress in the clay layer at pile 2:

$$\sigma_{z2} = \frac{P_1}{l^2} K_{z21} + \frac{P_2}{l^2} K_{z22} + \frac{P_3}{l^2} K_{z23} + \frac{P_4}{l^2} K_{z24} + \frac{P_5}{l^2} K_{z25} + \frac{P_6}{l^2} K_{z26}$$
$$\sigma_{z2} = \frac{500}{10^2}\left(0.5157 + 0.6335 + 0.5157 + 0.4487 + 0.5157 + 0.4487\right)$$
$$\sigma_{z2} = 15.39 \text{ [kN/m}^2\text{]}$$

Displacement of pile 2:

$$s_2 = \frac{1}{E_s}\sigma_{z2}h = \frac{15.39\times 2}{5000}$$

$$s_2 = 6.16\times 10^{-3} \text{ [m]}$$

Settlement of pile 2:

$$s_{P2} = s_2 + \Delta_1$$

$$s_{P2} = 6.16\times 10^{-3} + 2.36\times 10^{-3}$$

$$s_{P2} = 8.52\times 10^{-3} \text{ [m]} = 8.52 \text{ [mm]}$$

4.8.2 Example 8: Settlement of pile group under a square rigid pile cap

Figure 4.17 shows a group of piles under a square free standing rigid pile cap. Piles have a length of l = 15 [m], a diameter of φ=0.3 [m] and a Modulus of Elasticity of E_p=3×10^7 [kN/m^2]. The pile cap is loaded by a central vertical load of N = 4000 [kN]. Determine the pile group settlement due to a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] below the pile tips. The Modulus of Compressibility of the clay E_s=2000 [kN/m^2]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume uniform friction piles and suppose free standing rigid pile cap

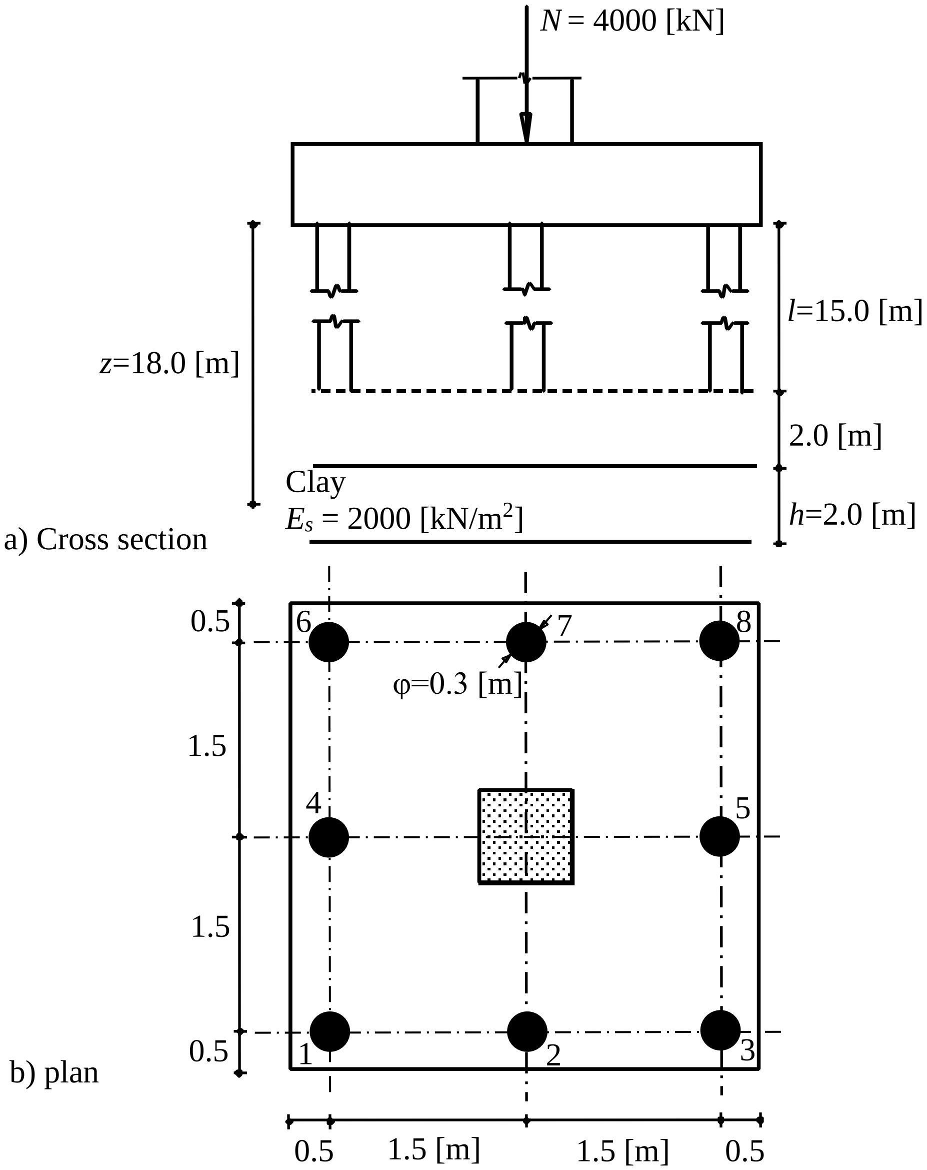

Figure 4.17 Pile cap with piles and the soil

Due to symmetry of pile group in geometry and load:

$$P_1 = P_3 = P_6 = P_8$$
$$P_2 = P_4 = P_5 = P_7$$

Due to the rigidity of the pile cap:

$$s_1 = s_2 = s_o \text{ and } P_1 \neq P_2$$

Elastic shortening of the pile:

$$\Delta = \frac{P\,l}{A_p E_p} = \frac{P_i 15}{\frac{\pi 0.3^2}{4} 3\times 10^7}$$
$$\Delta_i = 7.0736\times 10^{-6} P_i$$

Depth ratio is:

$$M = \frac{z}{l} = \frac{18}{15} = 1.2 \text{ [-]}$$

Determining radial distance r_{1j} between corner pile 1 and other piles (Figure 4.18):

$$r_{11} = 0$$
$$r_{12} = r_{14} = 1.5 \text{ [m]}$$
$$r_{13} = r_{16} = 3 \text{ [m]}$$
$$r_{15} = r_{17} = \sqrt{3^2 + 1.5^2} = 3.354 \text{ [m]}$$
$$r_{18} = \sqrt{3^2 + 3^2} = 4.243 \text{ [m]}$$

Radial distance ratios for pile 1:

$$N_{11} = 0$$
$$N_{12} = N_{14} = \frac{r_{12}}{l} = \frac{1.5}{15} = 0.1 \text{ [-]}$$
$$N_{13} = N_{16} = \frac{r_{13}}{l} = \frac{3}{15} = 0.2 \text{ [-]}$$
$$N_{15} = N_{17} = \frac{r_{15}}{l} = \frac{3.354}{15} = 0.22 \text{ [-]}$$
$$N_{18} = \frac{r_{18}}{l} = \frac{4.243}{15} = 0.28 \text{ [-]}$$

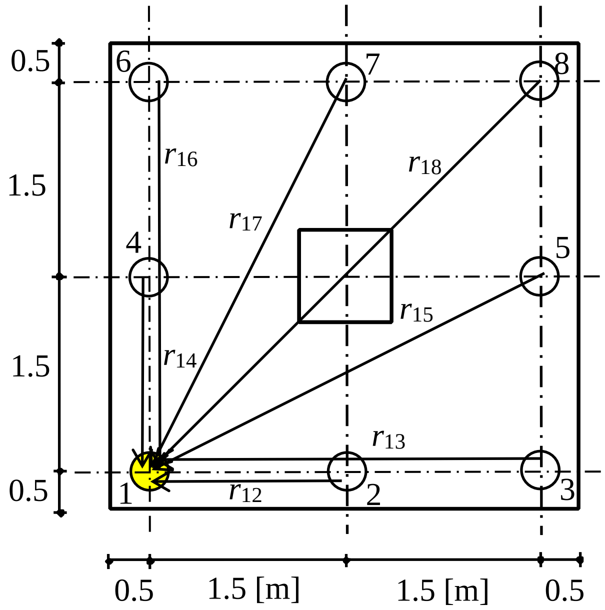

Figure 4.18 Radial distance r_{1j} between corner pile 1 and other piles

From Table 4.6, vertical stress coefficients K_{z1j} for pile 1 due to pile j are:

$$K_{z11} = 0.9440 \text{ [-]}$$

$$K_{z12} = K_{z14} = 0.7922 \text{ [-]}$$

$$K_{z13} = K_{z16} = 0.5588 \text{ [-]}$$

$$K_{z15} = K_{z17} = 0.5206 \text{ [-]}$$

$$K_{z18} = 0.3988 \text{ [-]}$$

Stress in the clay layer at pile 1:

$$\sigma_{z1} = \frac{P_1}{l^2}K_{z11} + \frac{P_2}{l^2}K_{z12} + \frac{P_3}{l^2}K_{z13} + \frac{P_4}{l^2}K_{z14} + \frac{P_5}{l^2}K_{z15} + \frac{P_6}{l^2}K_{z16} + \frac{P_7}{l^2}K_{z17} + \frac{P_8}{l^2}K_{z18}$$

$$\sigma_{z1} = \frac{P_1}{l^2}\left(K_{z11} + K_{z13} + K_{z16} + K_{z18}\right) + \frac{P_2}{l^2}\left(K_{z12} + K_{z14} + K_{z15} + K_{z17}\right)$$

$$\sigma_{z1} = \frac{P_1}{15^2}\left(0.9440 + 0.5588 + 0.5588 + 0.3988\right) + \frac{P_2}{15^2}\left(0.7922 + 0.7922 + 0.5206 + 0.5206\right)$$

$$\sigma_{z1} = 0.010935 P_1 + 0.011669 P_2$$

Displacement of pile 1:

$$s_1 = \frac{1}{E_s}\sigma_{z1}h = \frac{1}{2000}\left(0.010935P_1 + 0.011669P_2\right)\ 2$$
$$s_1 = \left(10.935P_1 + 11.669P_2\right)\times 10^{-6}$$

Elastic shortening of pile 1:

$$\Delta_1 = 7.0736\times 10^{-6}\,P_1$$

Settlement of pile 1:

$$s_{P1} = s_1 + \Delta_1$$
$$s_{P1} = \left(18.009P_1 + 11.669P_2\right)\times 10^{-6}$$

Determining radial distance r_{2j} between edge pile 2 and other piles (Figure 4.19):

$$r_{21} = r_{23} = 1.5\ [\text{m}]$$
$$r_{22} = 0$$
$$r_{24} = r_{25} = \sqrt{1.5^2 + 1.5^2} = 2.12\ [\text{m}]$$
$$r_{26} = r_{28} = \sqrt{3^2 + 1.5^2} = 3.354\ [\text{m}]$$
$$r_{27} = 3\ [\text{m}]$$

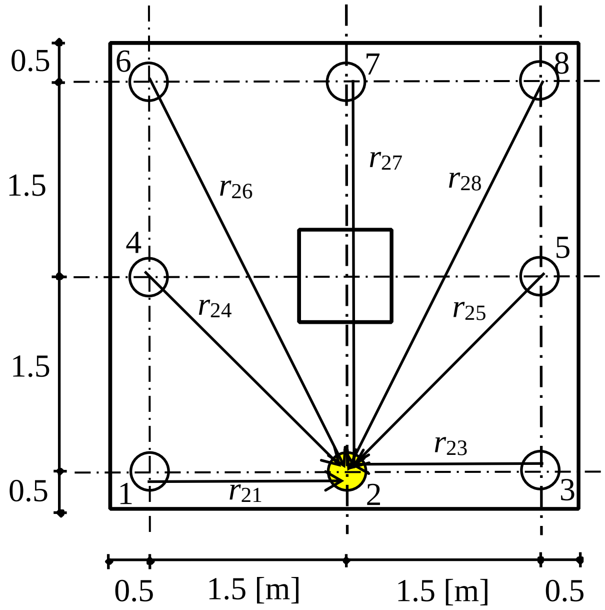

Figure 4.19 Radial distance r_{2j} between edge pile 2 and other piles

Radial distance ratios for pile 2:

$$N_{21} = N_{23} = \frac{r_{21}}{l} = \frac{1.5}{15} = 0.1 \text{ [-]}$$
$$N_{22} = 0 \text{ [-]}$$
$$N_{24} = N_{25} = \frac{r_{24}}{l} = \frac{2.12}{15} = 0.14 \text{ [-]}$$
$$N_{26} = N_{28} == \frac{r_{26}}{l} = \frac{3.354}{15} = 0.22 \text{ [-]}$$
$$N_{27} = \frac{r_{27}}{l} = \frac{3}{15} = 0.2 \text{ [-]}$$

From Table 4.6, vertical stress coefficients K_{z2j} for pile 2 due to pile j are:

$K_{z21} = K_{z23}$= 0.7922 [-]

K_{z22} = 0.9440 [-]

$K_{z24} = K_{z25}$ = 0.6929 [-]

$K_{z26} = K_{z28}$ = 0.5206 [-]

K_{z27} = 0.5588 [-]

Stress in the clay layer at pile 2:

$$\sigma_{z2} = \frac{P_1}{l^2} K_{z21} + \frac{P_2}{l^2} K_{z22} + \frac{P_3}{l^2} K_{z23} + \frac{P_4}{l^2} K_{z24} + \frac{P_5}{l^2} K_{z25} + \frac{P_6}{l^2} K_{z26} + \frac{P_7}{l^2} K_{z27} + \frac{P_8}{l^2} K_{z28}$$
$$\sigma_{z2} = \frac{P_{11}}{l^2}\left(K_{z21} + K_{z23} + K_{z26} + K_{z28}\right) + \frac{P_2}{l^2}\left(K_{z22} + K_{z24} + K_{z25} + K_{z27}\right)$$
$$\sigma_{z2} = \frac{P_1}{15^2}\left(0.7922 + 0.7922 + 0.5206 + 0.5206\right) + \frac{P_2}{15^2}\left(0.9440 + 0.6929 + 0.6929 + 0.5588\right)$$
$$\sigma_{z2} = 0.011669 P_1 + 0.012838 P_2$$

Displacement of pile 2:

$$s_2 = \frac{1}{E_s}\sigma_{z2} h = \frac{1}{2000}\left(0.011669 P_1 + 0.012838 P_2\right)\ 2$$
$$s_2 = \left(11.669 P_1 + 12.838 P_2\right) \times 10^{-6}$$

Elastic shortening of pile 1:

$$\Delta_2 = 7.0736 \times 10^{-6} P_2$$

Settlement of pile 1:

$$s_{P2} = s_2 + \Delta_2$$
$$s_{P2} = (11.669 P_1 + 19.912 P_2) \times 10^{-6}$$

Equating the settlement equations of pile 1 and 2 gives:

$$(18.009 P_1 + 11.669 P_2) \times 10^{-6} = (11.669 P_1 + 19.912 P_2) \times 10^{-6}$$

or

$$P_1 - 1.3 P_2 = 0$$

Equilibrium of the vertical forces gives:

$$\sum_1^8 P_i = N$$
$$P_1 + P_2 = 1000$$

Solving the above two equation of pile reactions gives:

$$P_1 = 565 \, [\text{kN}] \text{ and } P_2 = 435 \, [\text{kN}]$$

Rigid settlement is:

$$s_o = (18.009 P_1 + 11.669 P_2) \times 10^{-6}$$
$$s_o = (18.009 \times 565 + 11.669 \times 435) \times 10^{-6}$$
$$s_o = 0.0153 \text{ [m]} = 1.53 \text{ [cm]}$$

4.8.3 Example 9: Settlement of pile group under a circular pile cap

Figure 4.20 shows a group of piles under a circular free standing pile cap. The pile cap is loaded by a central vertical load of N = 4000 [kN]. Determine the pile group settlement for a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] below the pile tips. Piles have a length of l = 17 [m], a diameter of φ=0.3 [m] and a Modulus of Elasticity of E_p=3×10^7 [kN/m^2]. Modulus of Compressibility of the clay is E_s =1000 [kN/m^2]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume uniform friction piles.

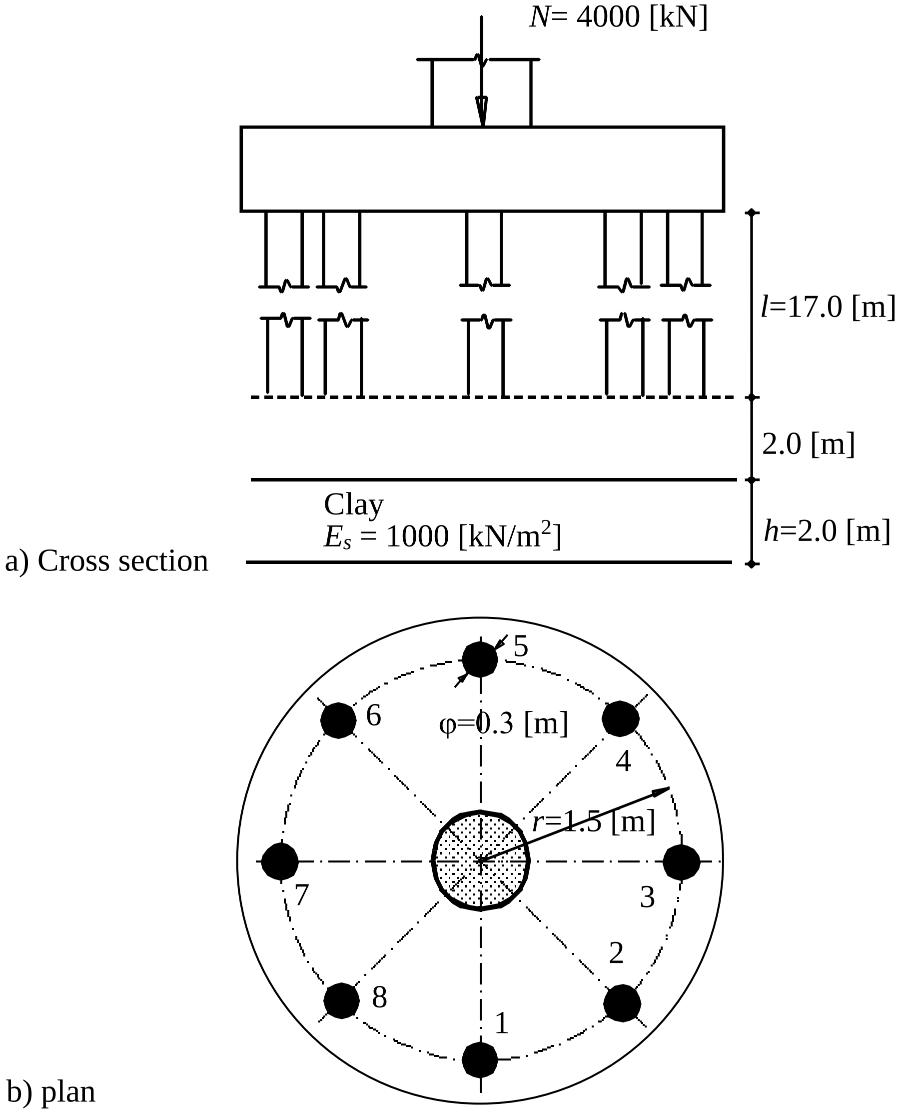

Figure 4.20 Pile cap with piles and the soil

For free standing pile cap and due to symmetry of piles about any axis:

$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = P_8 = \frac{4000}{8} = 500 \text{ [kN]}$$

Due to symmetry of pile group in geometry and load:

$$s_1 = s_2 = s_3 = s_4 = s_5 = s_6 = s_7 = s_8 = s_o$$

Elastic shortening of a single pile:

$$\Delta = \frac{P\,l}{A_p E_p} = \frac{500 \times 17}{\frac{\pi 0.3^2}{4} 3 \times 10^7}$$

$$\Delta = 4.01 \times 10^{-3} \text{ [m]}$$

$$\Delta = 0.401 \text{ [cm]}$$

Depth ratio is:

$$M = \frac{z}{l} = \frac{20}{17} = 1.176 \text{ [-]} \approx 1.2 \text{ [-]}$$

Determining radial distance r_{1j} between corner pile 1 and other piles (Figure 4.21):

$$r_{11} = 0$$

$$r_{12} = r_{18} = 1.2 \text{ [m]}$$

$$r_{13} = r_{17} = 2.12 \text{ [m]}$$

$$r_{14} = r_{16} = 2.8 \text{ [m]}$$

$$r_{15} = 3 \text{ [m]}$$

Radial distance ratios for pile 1:

$$N_{11} = 0$$

$$N_{12} = N_{18} = \frac{r_{12}}{l} = \frac{1.2}{17} = 0.07 \text{ [-]}$$

$$N_{13} = N_{17} = \frac{r_{13}}{l} = \frac{2.12}{17} = 0.124 \text{ [-]}$$

$$N_{14} = N_{16} = \frac{r_{14}}{l} = \frac{2.8}{17} = 0.165 \text{ [-]}$$

$$N_{15} = \frac{r_{15}}{l} = \frac{3}{17} = 0.176 \text{ [-]}$$

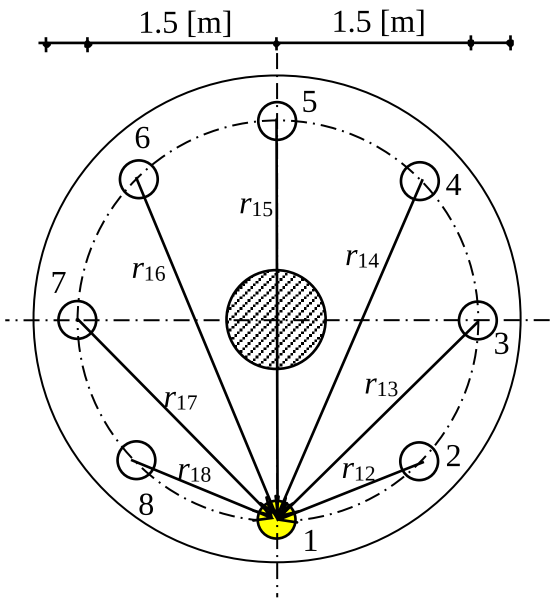

Figure 4.21 Radial distance r_{1j} between corner pile 1 and other piles

From Table 4.6, vertical stress coefficients K_{z1j} for pile 1 due to pile j are:

$$K_{z11} = 0.944 \text{ [-]}$$

$$K_{z12} = K_{z18} = 0.86 \text{ [-]}$$

$$K_{z13} = K_{z17} = 0.85 \text{ [-]}$$

$$K_{z14} = K_{z16} = 0.67 \text{ [-]}$$

$$K_{z15} = 0.624 \text{ [-]}$$

Stress in the clay layer at pile 1:

$$\sigma_{z1} = \frac{P_1}{l^2}K_{z11} + \frac{P_2}{l^2}K_{z12} + \frac{P_3}{l^2}K_{z13} + \frac{P_4}{l^2}K_{z14} + \frac{P_5}{l^2}K_{z15} + \frac{P_6}{l^2}K_{z16} + \frac{P_7}{l^2}K_{z17} + \frac{P_8}{l^2}K_{z18}$$

$$\sigma_{z1} = \frac{500}{17^2}(0.944 + 0.86 + 0.85 + 0.67 + 0.624 + 0.67 + 0.85 + 0.86)$$

$$\sigma_{z1} = 10.95 \text{ [kN/m}^2\text{]}$$

Displacement of pile 1:

$$s_1 = \frac{1}{E_s}\sigma_{z1}h = \frac{10.95 \times 2}{1000}$$

$$s_1 = 0.022 \text{ [m]}$$

$$s_1 = 2.2 \text{ [cm]}$$

Settlement of pile 1:

$$s_{P1} = s_1 + \Delta$$
$$s_{P1} = 2.2 + 0.401$$
$$s_{P1} = 2.601 \text{ [cm]}$$

4.8.4 Example 10: Settlement of pile group under a square rigid pile cap

Figure 4.22 shows a group of piles under a square free standing rigid pile cap. The pile cap is loaded by a vertical load of N = 4000 [kN] with an eccentricity e_x = 0.15 [m] in x-direction. Determine the pile group settlement for a clay layer of 2.0 [m] thickness and lies at a depth of 2.0 [m] below the pile tips. Piles have a length of l = 10 [m], a diameter of φ=0.3 [m] and a Modulus of Elasticity of E_p=3x10^7 [kN/m^2]. Modulus of Compressibility of the clay is E_s =2000 [kN/m^2]. *Poission's* ratio of the soil is ν_s = 0.3 [-]. Assume uniform friction piles.

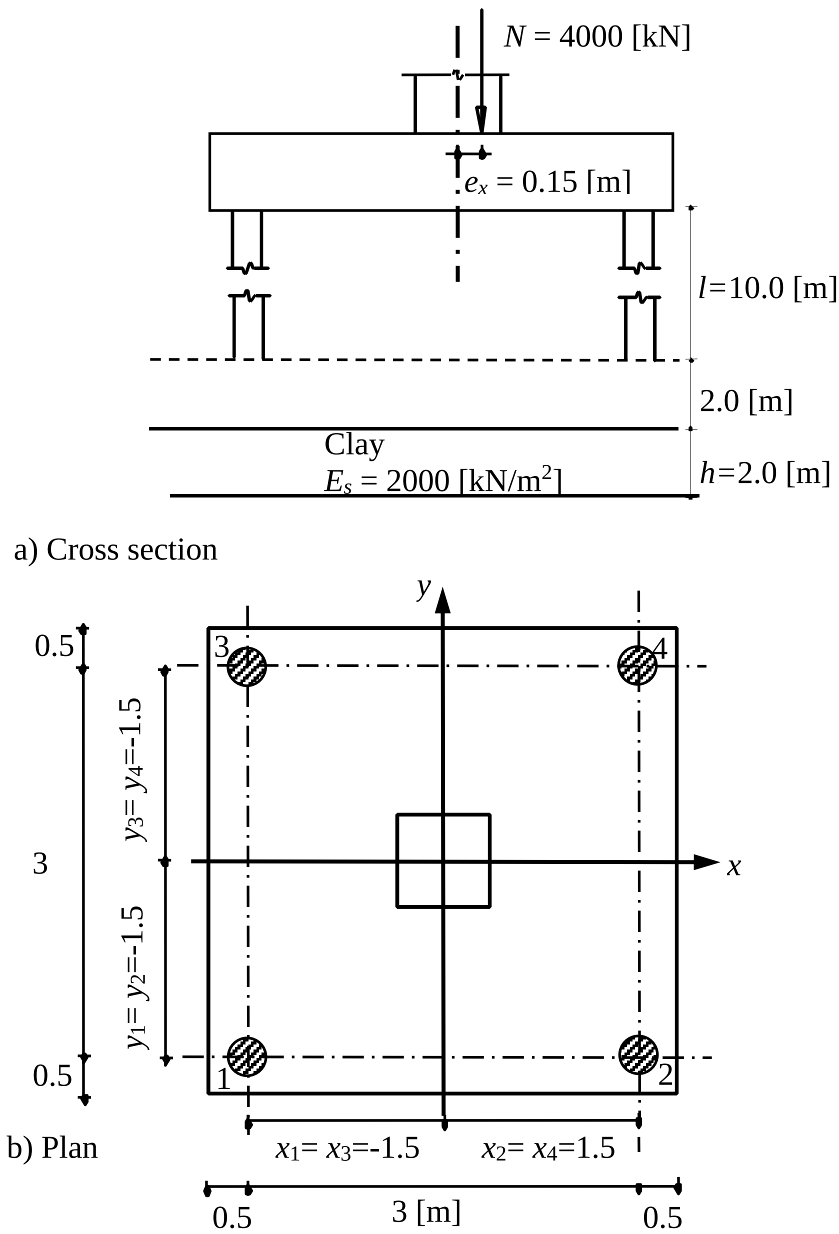

Figure 4.22 Pile cap with piles and the soil

4.8.4.1 Case of uniform settlement ($e_x = 0$ and $e_y = 0$)

For a rigid pile cap with a centric load (Figure 4.23), the settlement will be uniform ($s_i = s_o$) and the raft will not rotate ($\theta_x = \theta_y = 0$). Therefore, the unknowns of the problem are n pile reactions P_i and the rigid body translation s_o [m].

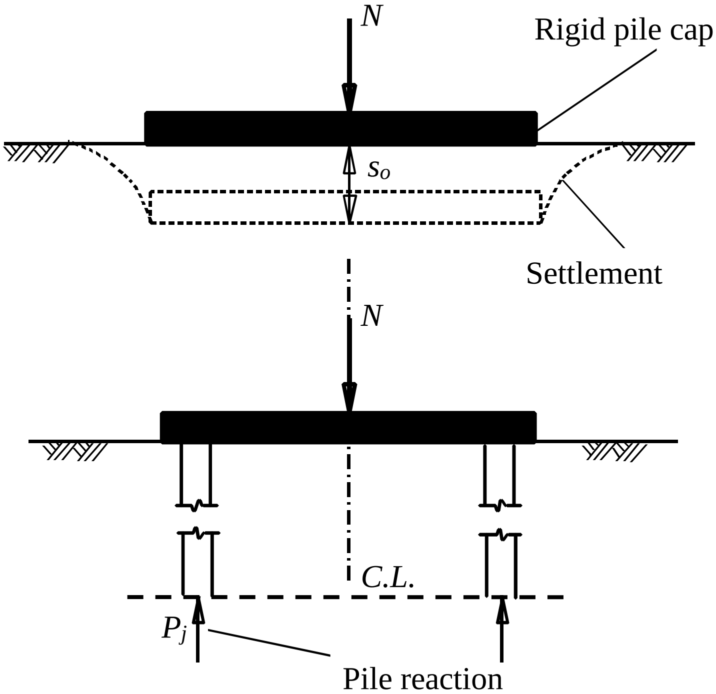

Figure 4.23 Settlement and pile reactions under a rigid pile cap (case of uniform settlement)

For free standing pile cap and due to symmetry of piles about any x- and y-axes:

$$P_1 = P_2 = P_3 = P_4 = \frac{4000}{4} = 1000 \text{ [kN]}$$

Due to symmetry of pile group in geometry and load:

$$s_1 = s_2 = s_3 = s_4 = s_o$$

Elastic shortening of a single pile:

$$\Delta = \frac{P\,l}{A_p E_p} = \frac{1000 \times 10}{\frac{\pi 0.3^2}{4} 3 \times 10^7}$$

$$\Delta = 4.72 \times 10^{-3} \text{ [m]}$$

$$\Delta = 0.472 \text{ [cm]}$$

Depth ratio is:

$$M = \frac{z}{l} = \frac{13}{10} = 1.3 \text{ [-]}$$

Determining radial distance r_{1j} between corner pile 1 and other piles (Figure 4.24):

$$r_{11} = 0$$
$$r_{12} = r_{14} = 3 \text{ [m]}$$
$$r_{13} = 4.24 \text{ [m]}$$

Radial distance ratios for pile 1:

$$N_{11} = 0$$
$$N_{12} = N_{14} = \frac{r_{12}}{l} = \frac{3}{10} = 0.3 \text{ [-]}$$
$$N_{13} = \frac{r_{13}}{l} = \frac{4.25}{10} = 0.425 \text{ [-]}$$

From Table 4.6, vertical stress coefficients K_{z1j} for pile 1 due to pile j are:

$$K_{z11} = 0.6335 \text{ [-]}$$

$$K_{z12} = K_{z13} = 0.33 \text{ [-]}$$

$$K_{z13} = 0.222 \text{ [-]}$$

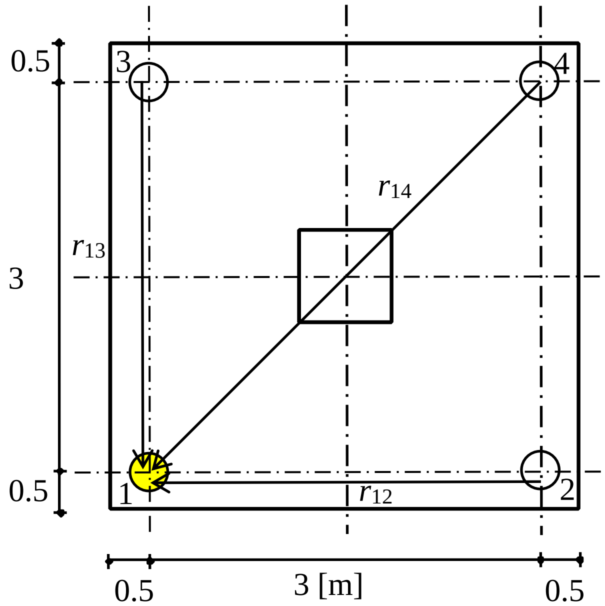

Figure 4.24 Radial distance r_{1j} between corner pile 1 and other piles

Stress in the clay layer at pile 1:

$$\sigma_{z1} = \frac{P_1}{l^2} K_{z11} + \frac{P_2}{l^2} K_{z12} + \frac{P_3}{l^2} K_{z13} + \frac{P_4}{l^2} K_{z14}$$

$$\sigma_{z1} = \frac{1000}{10^2} \left(0.6335 + 0.33 + 0.33 + 0.222\right)$$

$$\sigma_{z1} = 15.155 \ [\text{kN/m}^2]$$

Displacement of pile 1:

$$s_1 = \frac{1}{E_s} \sigma_{z1} h = \frac{15.155 \times 2}{2000}$$

$$s_1 = 0.0152 [\text{m}]$$

$$s_1 = 1.52 [\text{cm}]$$

Settlement of pile 1:

$$s_{P1} = s_1 + \Delta$$

$$s_{P1} = 1.52 + 0.472$$

$$s_{P1} = 1.992 \ [\text{cm}] = s_o$$

4.8.4.2 Case of single eccentric load ($e_x \neq 0$)

For a rigid pile cap with a single eccentric load about *y*-axis (Figure 4.25), the unknowns of the problem are *n* pile reactions P_i, the uniform rigid body translation s_o and the rotation θ_y [Rad] about *y*-axis. The uniform rigid body translation s_o is obtained by analyzing the same rigid pile cap without eccentricity.

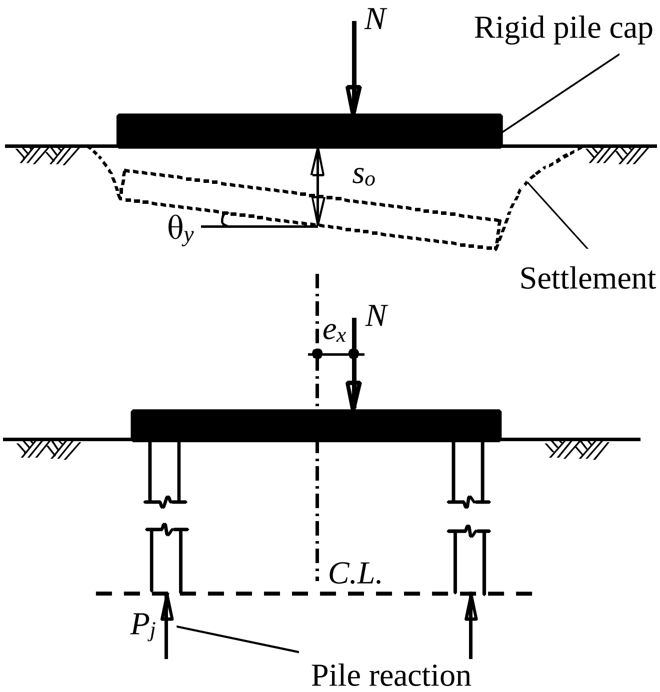

Figure 4.25 Settlement and pile reactions under a rigid pile cap (case of a single eccentric load)

For free standing pile cap and due to symmetry of piles about *x*-axes:

$$P_1 = P_3 \text{ and } P_2 = P_4$$

also:

$$s_1 = s_3 \text{ and } s_2 = s_4$$

For overall equilibrium of the vertical forces:

$$P_1 + P_2 + P_3 + P_4 = N$$
$$P_1 + P_2 = \frac{4000}{2} = 2000 \text{ [kN]} \tag{4.11}$$

Equating the moment due to resultant N about the y-axis by the sum of moments due to pile reactions P_i about that axis, gives:

$$P_1 \; x_1 + P_2 \; x_2 + P_3 \; x_3 + P_4 \; x_4 = N \; e_x$$
$$-3P_1 + 3P_2 = 4000 \times 0.15 \qquad (4.12)$$
$$-P_1 + P_2 = 200$$

Solving Eqns (4.11) and (4.12) together, gives:

$$P_1 = P_3 = 900\,[\text{kN}] \text{ and } P_2 = P_4 = 1100\,[\text{kN}]$$

Stress in the clay layer at pile 1:

$$\sigma_{z1} = \frac{P_1}{l^2} K_{z11} + \frac{P_2}{l^2} K_{z12} + \frac{P_3}{l^2} K_{z13} + \frac{P_4}{l^2} K_{z14}$$
$$\sigma_{z1} = \frac{900}{10^2}(0.6335 + 0.33) + \frac{1100}{10^2}(0.33 + 0.222)$$
$$\sigma_{z1} = 14.744 \; [\text{kN/m}^2]$$

Displacement of pile 1:

$$s_1 = \frac{1}{E_s} \sigma_{z1} h = \frac{14.744 \times 2}{2000}$$
$$s_1 = 0.0147\,[\text{m}]$$
$$s_1 = 1.47\,[\text{cm}]$$

Settlement of pile 1:

$$s_{P1} = s_1 + \Delta$$
$$s_{P1} = 1.47 + 0.472$$
$$s_{P1} = 1.946 \; [\text{cm}] = s_{P3}$$

Determining radial distance r_{1j} between corner pile 2 and other piles (Figure 4.26):

$$r_{22} = 0$$
$$r_{21} = r_{24} = 3 \; [\text{m}]$$
$$r_{23} = 4.24 \; [\text{m}]$$

Radial distance ratios for pile 2:

$$N_{22} = 0$$

$$N_{21} = N_{24} = \frac{r_{21}}{l} = \frac{3}{10} = 0.3 \text{ [-]}$$

$$N_{23} = \frac{r_{23}}{l} = \frac{4.25}{10} = 0.425 \text{ [-]}$$

From Table 4.6, vertical stress coefficients K_{z1j} for pile 1 due to pile j are:

$$K_{z22} = 0.6335 \text{ [-]}$$

$$K_{z21} = K_{z24} = 0.33 \text{ [-]}$$

$$K_{z23} = 0.222 \text{ [-]}$$

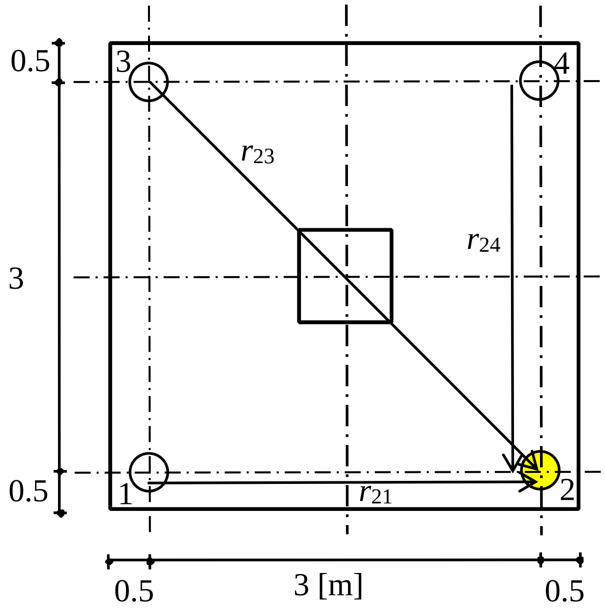

Figure 4.26 Radial distance r_{1j} between corner pile 2 and other piles

Stress in the clay layer at pile 2:

$$\sigma_{z2} = \frac{P_1}{l^2} K_{z21} + \frac{P_2}{l^2} K_{z22} + \frac{P_3}{l^2} K_{z23} + \frac{P_4}{l^2} K_{z24}$$

$$\sigma_{z2} = \frac{900}{10^2}(0.33 + 0.222) + \frac{1100}{10^2}(0.6335 + 0.33)$$

$$\sigma_{z2} = 15.567 \text{ [kN/m}^2\text{]}$$

Displacement of pile 2:

$$s_2 = \frac{1}{E_s}\sigma_{z2}h = \frac{15.567 \times 2}{2000}$$
$$s_2 = 0.0156\,[\mathrm{m}]$$
$$s_2 = 1.56\,[\mathrm{cm}]$$

Settlement of pile 2:

$$s_{P2} = s_1 + \Delta$$
$$s_{P2} = 1.56 + 0.472$$
$$s_{P2} = 2.03\ [\mathrm{cm}] = s_{P4}$$

Due to the pile cap rigidity, the following linear relation expresses the settlement s_i at pile i that has a distance x_i from the geometry centroid:

$$s_i = s_o + x_i \tan \theta_y \tag{4.13}$$

Applying the above equation on pile 2, gives the rotation θ_y [Rad] about y-axis as follows:

$$s_{P2} = s_o + x_2 \tan \theta_y$$
$$2.03 = 1.992 + 1.5 \tan \theta_y$$

Then,

$$\theta_y = 1.45\ [\mathrm{Deg}]$$

Table 4.6 Stress coefficients according to *GEDDES* (*Poisso*'s ratio $\nu_s = 0.3$ [-])

Stress coefficients for constant skin friction

M/N	0.00	0.02	0.04	0.06	0.08	0.10	0.20	0.50	1.00	2.00
1.0		6.8154	3.4044	2.2673	1.6984	1.3567	0.6695	0.2346	0.0686	0.0076
1.1	1.9240	1.8595	1.7074	1.5137	1.3210	1.1503	0.6419	0.2335	0.0728	0.0091
1.2	0.9440	0.9384	0.9164	0.8824	0.8399	0.7922	0.5588	0.2292	0.0760	0.0105
1.3	0.6335	0.6168	0.6102	0.5993	0.5848	0.5675	0.4598	0.2207	0.0782	0.0120
1.4	0.4618	0.4535	0.4507	0.4460	0.4395	0.4315	0.3761	0.2082	0.0796	0.0134
1.5	0.3421	0.3548	0.3532	0.3508	0.3474	0.3431	0.3115	0.1934	0.0800	0.0148
1.6	0.2901	0.2885	0.2878	0.2863	0.2842	0.2816	0.2621	0.1777	0.0796	0.0160
1.7	0.2263	0.2413	0.2406	0.2397	0.2384	0.2367	0.2239	0.1623	0.0784	0.0172
1.8	0.1954	0.2056	0.2053	0.2046	0.2038	0.2026	0.1937	0.1479	0.0766	0.0182
1.9	0.1828	0.1780	0.1779	0.1774	0.1767	0.1760	0.1696	0.1347	0.0744	0.0191
2.0	0.1222	0.1559	0.1560	0.1556	0.1551	0.1545	0.1498	0.1229	0.0718	0.0199

Stress coefficients for a point load (end bearing)

M/N	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.50	2.00
1.0		0.1014	0.0988	0.0946	0.0890	0.0825	0.0642	0.0464	0.0210	0.0087
1.1	19.4008	3.9108	0.5987	0.2126	0.1289	0.0988	0.0669	0.0476	0.0223	0.0097
1.2	4.9155	2.9316	1.0373	0.4007	0.2030	0.1305	0.0723	0.0493	0.0235	0.0106
1.3	2.2250	1.7491	0.9770	0.4977	0.2721	0.1689	0.0809	0.0520	0.0247	0.0116
1.4	1.2793	1.1167	0.7816	0.4897	0.3036	0.1977	0.0910	0.0555	0.0260	0.0125
1.5	0.8389	0.7696	0.6078	0.4362	0.3016	0.2101	0.1000	0.0595	0.0274	0.0134
1.6	0.5976	0.5634	0.4774	0.3743	0.2813	0.2089	0.1064	0.0632	0.0288	0.0143
1.7	0.4506	0.4318	0.3824	0.3182	0.2542	0.1991	0.1096	0.0662	0.0302	0.0152
1.8	0.3540	0.3429	0.3126	0.2710	0.2265	0.1852	0.1098	0.0683	0.0315	0.0161
1.9	0.2870	0.2799	0.2603	0.2324	0.2009	0.1699	0.1077	0.0694	0.0327	0.0170
2.0	0.2383	0.2336	0.2204	0.2010	0.1783	0.1549	0.1041	0.0695	0.0337	0.0177

Stress coefficients for linear variation of skin friction

M/N	0.02	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.50	2.00
1.0	12.128	1.8346	0.7276	0.4126	0.2748	0.1997	0.1066	0.0616	0.0214	0.0077
1.1	2.961	1.6430	0.7679	0.4434	0.2937	0.2115	0.1118	0.0654	0.0238	0.0090
1.2	1.346	1.0863	0.6899	0.4420	0.3025	0.2198	0.1166	0.0689	0.0262	0.0104
1.3	0.825	0.7408	0.5639	0.4067	0.2957	0.2212	0.1201	0.0720	0.0284	0.0117
1.4	0.577	0.5407	0.4530	0.3573	0.2765	0.2150	0.1219	0.0744	0.0304	0.0130
1.5	0.435	0.4160	0.3679	0.3084	0.2515	0.2033	0.1215	0.0760	0.0322	0.0143
1.6	0.343	0.3326	0.3040	0.2657	0.2256	0.1887	0.1192	0.0768	0.0338	0.0155
1.7	0.280	0.2739	0.2556	0.2299	0.2012	0.1731	0.1153	0.0767	0.0351	0.0166
1.8	0.234	0.2304	0.2182	0.2003	0.1795	0.1580	0.1104	0.0758	0.0361	0.0176
1.9	0.200	0.1972	0.1887	0.1759	0.1604	0.1439	0.1048	0.0742	0.0368	0.0186
2.0	0.175	0.1714	0.1651	0.1556	0.1439	0.1310	0.0989	0.0721	0.0373	0.0194
2.1	0.155	0.1504	0.1458	0.1387	0.1297	0.1195	0.0931	0.0697	0.0375	0.0200
2.2	0.135	0.1334	0.1299	0.1244	0.1173	0.1092	0.0873	0.0670	0.0375	0.0206
2.3	0.120	0.1193	0.1165	0.1122	0.1066	0.1000	0.0819	0.0642	0.0373	0.0210
2.4	0.106	0.1075	0.1052	0.1018	0.0972	0.0919	0.0767	0.0614	0.0369	0.0214
2.5	0.096	0.0974	0.0955	0.0927	0.0890	0.0847	0.0718	0.0586	0.0363	0.0216

Table 4.7 Stress coefficients according to *GEDDES* (*Poisso*'s ratio $\nu_s = 0.2$ [-])

Stress coefficients for constant skin friction

M/N	0,00	0,02	0,04	0,06	0,08	0,10	0,20	0,50	1,00	2,00
1,0		6,4708	3,2375	2,1595	1,6202	1,2963	0,6445	0,2300	0,0690	0,0081
1,1	1,7962	1,7330	1,5946	1,4181	1,2418	1,0850	0,6138	0,2283	0,0730	0,0096
1,2	0,8827	0,8778	0,8577	0,8268	0,7881	0,7446	0,5307	0,2231	0,0759	0,0111
1,3	0,5828	0,5789	0,5729	0,5630	0,5498	0,5340	0,4355	0,2138	0,0779	0,0125
1,4	0,4398	0,4269	0,4244	0,4200	0,4141	0,4068	0,3562	0,2010	0,0789	0,0139
1,5	0,3239	0,3347	0,3334	0,3312	0,3281	0,3241	0,2952	0,1862	0,0790	0,0152
1,6	0,2735	0,2729	0,2722	0,2708	0,2690	0,2666	0,2487	0,1708	0,0784	0,0165
1,7	0,2216	0,2287	0,2281	0,2272	0,2261	0,2246	0,2127	0,1559	0,0770	0,0175
1,8	0,1896	0,1953	0,1950	0,1944	0,1936	0,1925	0,1844	0,1420	0,0750	0,0185
1,9	0,1772	0,1694	0,1692	0,1688	0,1682	0,1675	0,1616	0,1293	0,0727	0,0193
2,0	0,1386	0,1485	0,1486	0,1483	0,1479	0,1473	0,1429	0,1180	0,0700	0,0201

Stress coefficients for a point load (end bearing)

M/N	0,00	0,10	0,20	0,30	0,40	0,50	0,75	1,00	1,50	2,00
1,0		0,0962	0,0937	0,0898	0,0847	0,0786	0,0615	0,0448	0,0208	0,0089
1,1	17,9768	3,7805	0,6196	0,2241	0,1334	0,1000	0,0660	0,0468	0,0223	0,0100
1,2	4,5562	2,7496	1,0019	0,3993	0,2059	0,1327	0,0725	0,0491	0,0236	0,0110
1,3	2,0636	1,6309	0,9246	0,4804	0,2676	0,1683	0,0812	0,0521	0,0250	0,0119
1,4	1,1874	1,0396	0,7340	0,4659	0,2930	0,1933	0,0906	0,0556	0,0263	0,0129
1,5	0,7792	0,7163	0,5690	0,4120	0,2879	0,2028	0,0986	0,0593	0,0277	0,0138
1,6	0,5556	0,5245	0,4464	0,3522	0,2668	0,1999	0,1040	0,0626	0,0291	0,0147
1,7	0,4194	0,4023	0,3574	0,2988	0,2402	0,1896	0,1063	0,0652	0,0304	0,0156
1,8	0,3299	0,3197	0,2922	0,2543	0,2136	0,1757	0,1059	0,0669	0,0316	0,0164
1,9	0,2677	0,2612	0,2434	0,2180	0,1892	0,1609	0,1034	0,0676	0,0326	0,0172
2,0	0,2225	0,2183	0,2062	0,1885	0,1678	0,1464	0,0996	0,0674	0,0334	0,0180

Stress coefficients for linear variation of skin friction

M/N	0,00	0,02	0,04	0,06	0,08	0,10	0,20	0,50	1,00	2,00
1,0		11,5296	5,3126	3,3017	2,3261	1,7582	0,7033	0,1963	0,0618	0,0082
1,1	2,8098	2,7516	2,4900	2,1590	1,8326	1,5466	0,7345	0,2074	0,0656	0,0096
1,2	1,2974	1,2526	1,2167	1,1616	1,0926	1,0160	0,6528	0,2141	0,0689	0,0110
1,3	0,7361	0,7692	0,7583	0,7415	0,7193	0,6926	0,5311	0,2139	0,0717	0,0123
1,4	0,4345	0,5391	0,5341	0,5272	0,5177	0,5059	0,4261	0,2068	0,0738	0,0136
1,5	0,4746	0,4059	0,4043	0,4007	0,3959	0,3898	0,3460	0,1947	0,0750	0,0148
1,6	0,3759	0,3222	0,3202	0,3184	0,3157	0,3122	0,2861	0,1802	0,0754	0,0160
1,7	0,2973	0,2631	0,2624	0,2611	0,2594	0,2574	0,2408	0,1651	0,0750	0,0170
1,8	0,2346	0,2219	0,2200	0,2195	0,2184	0,2169	0,2058	0,1506	0,0739	0,0180
1,9	0,0952	0,1894	0,1881	0,1879	0,1870	0,1860	0,1782	0,1371	0,0722	0,0188
2,0	0,3421	0,1658	0,1629	0,1629	0,1624	0,1618	0,1561	0,1248	0,0700	0,0196

Table 4.8 Stress coefficients according to *GEDDES* (*Poisso*'s ratio ν_s = 0.4 [-])

Stress coefficients for constant skin friction

M/N	0,00	0,02	0,04	0,06	0,08	0,10	0,20	0,50	1,00	2,00
1,0		7,2748	3,6271	2,4110	1,8026	1,4373	0,7029	0,2407	0,0681	0,0069
1,1	2,1022	2,0282	1,8576	1,6411	1,4266	1,2373	0,6794	0,2404	0,0725	0,0083
1,2	1,0231	1,0194	0,9946	0,9566	0,9089	0,8556	0,5964	0,2373	0,0760	0,0098
1,3	0,6830	0,6675	0,6600	0,6478	0,6316	0,6121	0,4921	0,2298	0,0787	0,0113
1,4	0,5015	0,4889	0,4859	0,4806	0,4733	0,4644	0,4026	0,2178	0,0805	0,0128
1,5	0,3674	0,3814	0,3797	0,3769	0,3731	0,3683	0,3331	0,2029	0,0813	0,0142
1,6	0,3174	0,3093	0,3085	0,3068	0,3045	0,3017	0,2799	0,1868	0,0812	0,0155
1,7	0,2534	0,2581	0,2574	0,2563	0,2549	0,2530	0,2387	0,1708	0,0803	0,0167
1,8	0,2021	0,2195	0,2191	0,2183	0,2173	0,2161	0,2063	0,1557	0,0787	0,0178
1,9	0,2078	0,1895	0,1894	0,1888	0,1882	0,1873	0,1802	0,1419	0,0766	0,0188
2,0	0,1376	0,1657	0,1658	0,1654	0,1649	0,1642	0,1590	0,1294	0,0741	0,0196

Stress coefficients for a point load (end bearing)

M/N	0,00	0,10	0,20	0,30	0,40	0,50	0,75	1,00	1,50	2,00
1,0		0,1084	0,1055	0,1009	0,0949	0,0877	0,0677	0,0484	0,0212	0,0084
1,1	21,2994	4,0844	0,5707	0,1972	0,1230	0,0971	0,0681	0,0487	0,0223	0,0093
1,2	5,3945	3,1742	1,0844	0,4025	0,1991	0,1276	0,0721	0,0496	0,0233	0,0102
1,3	2,4403	1,9066	1,0469	0,5207	0,2780	0,1698	0,0805	0,0519	0,0244	0,0111
1,4	1,4020	1,2195	0,8450	0,5215	0,3178	0,2035	0,0914	0,0555	0,0257	0,0120
1,5	0,9184	0,8407	0,6596	0,4685	0,3198	0,2199	0,1018	0,0597	0,0270	0,0129
1,6	0,6535	0,6151	0,5188	0,4038	0,3005	0,2208	0,1096	0,0639	0,0285	0,0138
1,7	0,4922	0,4711	0,4157	0,3440	0,2728	0,2119	0,1139	0,0676	0,0300	0,0147
1,8	0,3863	0,3738	0,3398	0,2933	0,2437	0,1979	0,1150	0,0702	0,0315	0,0157
1,9	0,3127	0,3048	0,2829	0,2516	0,2164	0,1820	0,1134	0,0718	0,0328	0,0166
2,0	0,2594	0,2541	0,2393	0,2176	0,1922	0,1662	0,1100	0,0723	0,0340	0,0174

Stress coefficients for linear variation of skin friction

M/N	0,00	0,02	0,04	0,06	0,08	0,10	0,20	0,50	1,00	2,00
1,0		12,9279	5,9279	3,6675	2,5727	1,9365	0,7600	0,2042	0,0614	0,0069
1,1	3,3296	3,2425	2,9197	2,5138	2,1167	1,7716	0,8125	0,2170	0,0652	0,0083
1,2	1,5192	1,4712	1,4267	1,3587	1,2739	1,1799	0,7393	0,2274	0,0689	0,0096
1,3	0,8791	0,8996	0,8862	0,8654	0,8380	0,8051	0,6076	0,2308	0,0723	0,0109
1,4	0,5115	0,6281	0,6216	0,6132	0,6014	0,5869	0,4890	0,2259	0,0752	0,0123
1,5	0,5289	0,4708	0,4686	0,4643	0,4583	0,4509	0,3971	0,2147	0,0773	0,0136
1,6	0,4059	0,3717	0,3698	0,3674	0,3641	0,3599	0,3279	0,1999	0,0786	0,0149
1,7	0,3846	0,3030	0,3016	0,3003	0,2981	0,2957	0,2753	0,1838	0,0788	0,0161
1,8	0,3117	0,2543	0,2520	0,2515	0,2502	0,2483	0,2347	0,1679	0,0782	0,0172
1,9	0,1267	0,2159	0,2146	0,2147	0,2135	0,2123	0,2027	0,1530	0,0769	0,0182
2,0	0,3438	0,1890	0,1853	0,1854	0,1848	0,1841	0,1771	0,1393	0,0749	0,0191

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