Example 2: Design of a square raft for different soil models and codes

1 Description of the problem

Many soil models are used for analysis of raft foundations. Each model gives internal forces for the raft different from that of the others. However, all models are considered save and correct. This example is carried out to show the differences in the design results when the raft is analyzed by different soil models.

A square raft has dimensions of 10 [m] × 10 [m]. The raft carries four symmetrical loads, each 1200 [kN] as shown in Figure 21. Column sides are 0.50 [m] × 0.50 [m], while column reinforcement is 8 Φ 19. To carry out the comparison of different codes and soil models, the raft thickness is chosen d = 0.6 [m] for all soil models and design codes. The raft rests on a homogeneous soil layer of thickness 10 [m] equal to the raft length, overlying a rigid base. The Modulus of Compressibility of the soil layer is E_s = 10 000 [kN/ m^2], while *Poisson's* ratio of the soil is ν_s = 0.3 [-].

The three subsoil models: Simple assumption model, *Winkler's* model and Continuum model (Isotropic elastic half-space soil medium and Layered soil medium) are represented by four mathematical calculation methods that are available in *ELPLA* (Table 12).

Table 12 Calculation methods

Method No.	Method
1	Linear contact pressure (Simple assumption model)
2	Constant modulus of subgrade reaction (*Winkler's* model)
5	Modulus of compressibility method for elastic raft on half-space soil medium (Isotropic elastic half-space soil medium - Continuum model)
7	Modulus of compressibility method for elastic raft on layered soil medium (Solving system of linear equations by elimination) (Layered soil medium - Continuum model)

2 Properties of raft material and section

2.1 Material properties

Young's modulus of concrete E_b = 34 000 000 [kN/ m^2]
Poisson's ratioof concrete ν_b = 0.20 [-]
Unit weight of concrete γ_b = 25 [kN/ m^3]

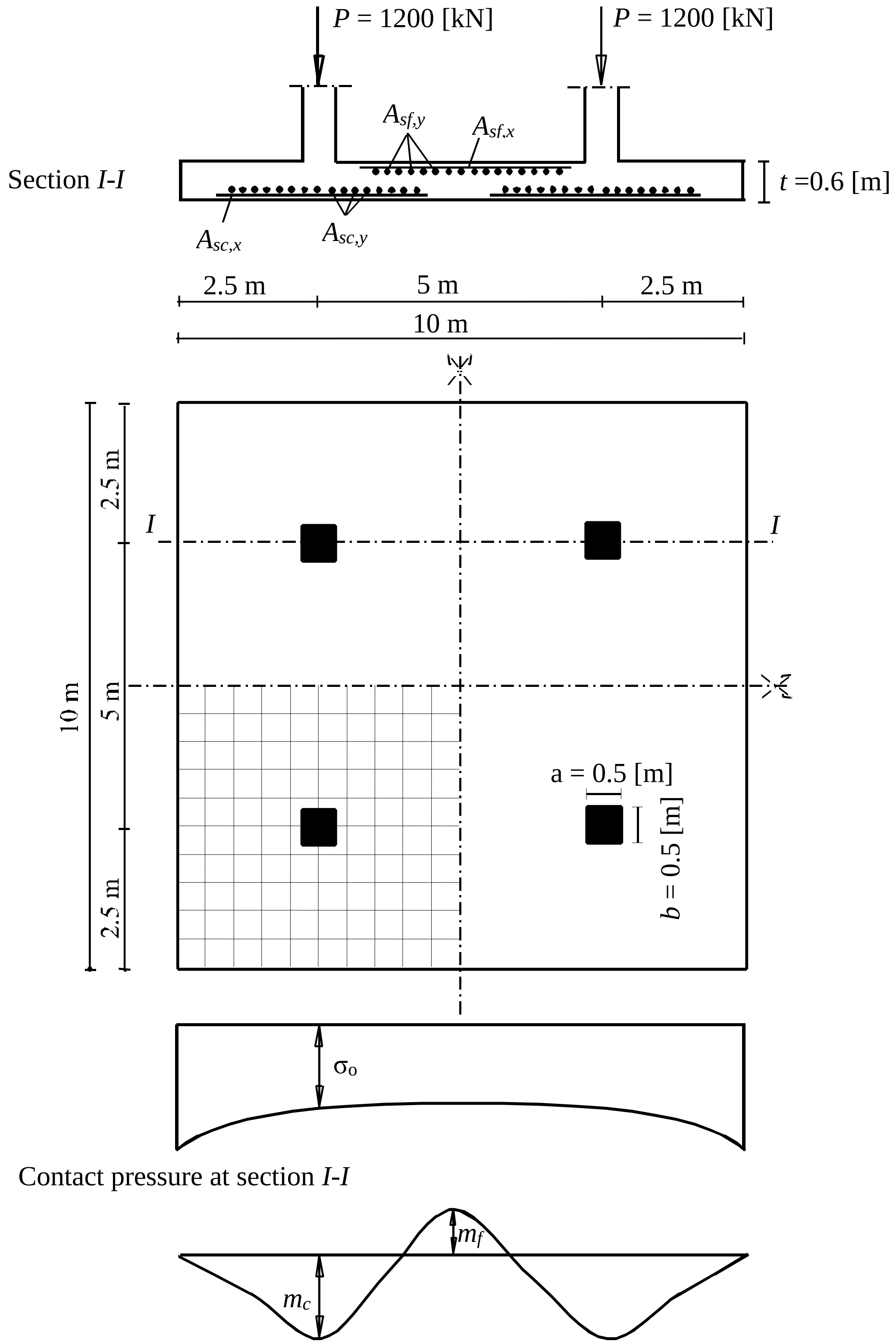

Figure 21 Raft dimensions with mesh and loads

2.2 Section properties

Width of the section to be designed	b = 1.0 [m]
Section thickness	t = 0.6 [m]
Concrete cover + 1/2 bar diameter	c = 5 [cm]
Effective depth of the section	$d = t - c$ = 0.55 [m]
Steel bar diameter	Φ = 22 [mm]
Minimum area of steel	$A_s\ min$ = 5 Φ 19 = 14.2 [cm^2/ m]

3 Analysis of the raft

The raft is subdivided into 400 square elements. Each has dimensions of 0.50 [m] × 0.50 [m] yielding to 21 × 21 nodal points for the raft and soil. Taking advantage of the symmetry in shape and load geometry about *x*- and *y*-axes, the analysis is carried out considering only a quarter of the raft. Because of the raft symmetry, the design is carried out only for section *I-I*.

Table 13 shows the contact pressure under the column σ_o, field moment m_f and the column moment m_c at the critical section *I-I* by application of different soil models. For the different codes, the raft is designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained. Finally, a comparison of the results of the two codes EC2 and DIN 1045 with soil models is presented.

Table 13 Contact pressure σ_o under the column, field moment m_f and column moment m_c at the critical section *I-I* by application of different soil models

Soil model		σ_o [kN/ m^2]	m_c [kN.m/ m]	m_f [kN.m/ m]
Simple assumption model	1	63	400	-13
Winkler's model	2	62	399	-15
Isotropic elastic half-space medium	5	42	504	136
Layered medium	7	45	492	111

4 Design for EC 2

4.1 Design for flexure moment

Material

Concrete grade	C 30/37
Steel grade	BSt 500
Characteristic compressive cylinder strength of concrete	f_{ck} = 30 [MN/ m^2]
Characteristic tensile yield strength of reinforcement	$f_{yk} = f_y$ = 500 [MN/ m^2]
Partial safety factor for concrete strength	γ_c = 1.5
Design concrete compressive strength	$f_{cd} = f_{ck}/\gamma_c$ = 30/1.5 = 20 [MN/ m^2]
Partial safety factor for steel strength	γ_s = 1.15
Design tensile yield strength of reinforcing steel	$f_{yd} = f_{yk}/\gamma_s$ = 500/1.15 = 435 [MN/ m^2]

Factored moment

Total load factor for both dead and live loads	γ = 1.5
Factored column moment	$M_{sd} = \gamma m_c$
Factored field moment	$M_{sd} = \gamma m_f$

Geometry

Effective depth of the section	d = 0.55 [m]
Width of the section to be designed	b = 1.0 [m]

Determination of tension reinforcement

The design of sections is carried out for EC 2 in table forms. Table 14 andTable 15 show the design of section *I-I*.

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 0.55^2(0.85 \times 20)} = 0.195 M_{sd}$$

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.195 M_{sd}} = 1 - \sqrt{1 - 0.39 M_{sd}}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega\left(\frac{(0.85 f_{cd})bd}{f_{yd}}\right)$$

$$A_s = \omega\left(\frac{(0.85 \times 20) \times 1.0 \times 0.55}{435}\right) = 0.021494\omega\,[\mathrm{m}^2/\,\mathrm{m}]$$

$$A_s = 214.943\omega\ [\mathrm{cm}^2/\,\mathrm{m}]$$

Table 14 Required bottom reinforcement under the column A_{sc} for different soil models

Soil model		M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{sc} [cm²/ m]
Simple assumption model	1	0.600	0.117	0.124	26.76
Winkler's model	2	0.599	0.116	0.124	26.70
Isotropic elastic half-space medium	5	0.757	0.147	0.160	34.39
Layered medium	7	0.737	0.143	0.156	33.43

Table 15 Required top reinforcement in the field A_{sf} for different soil models

Soil model		M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{sf} [cm²/ m]
Simple assumption model	1	0.0197	0.0038	0.0038	0.83
Winkler's model	2	0.0223	0.0043	0.0044	0.94
Isotropic elastic half-space medium	5	-	-	-	-
Layered medium	7	-	-	-	-

Chosen reinforcement

Table 16 shows the number of steel bars under the column and in the field between columns at section *I-I* considering different soil models. The chosen diameter of steel bars is Φ = 22 [mm].

Table 16 Chosen reinforcement at section *I-I* for different soil models

Soil model		Chosen reinforcement	
		Bottom *Rft* under column A_{sc}	Top *Rft* in the field A_{sf}
Simple assumption model	1	8 Φ 22 = 30.40 [cm^2/ m]	*min* A_s
Winkler=s model	3	8 Φ 22 = 30.40 [cm^2/ m]	*min* A_s
Isotropic elastic half-space medium	5	10 Φ 22 = 38.00 [cm^2/ m]	*min* A_s
Layered medium	7	9 Φ 22 = 34.20 [cm^2/ m]	*min* A_s

4.2 Check for punching shear

The critical section for punching shear is at a distance r = 0.825 [m] around the circumference of the column as shown in Figure 22.

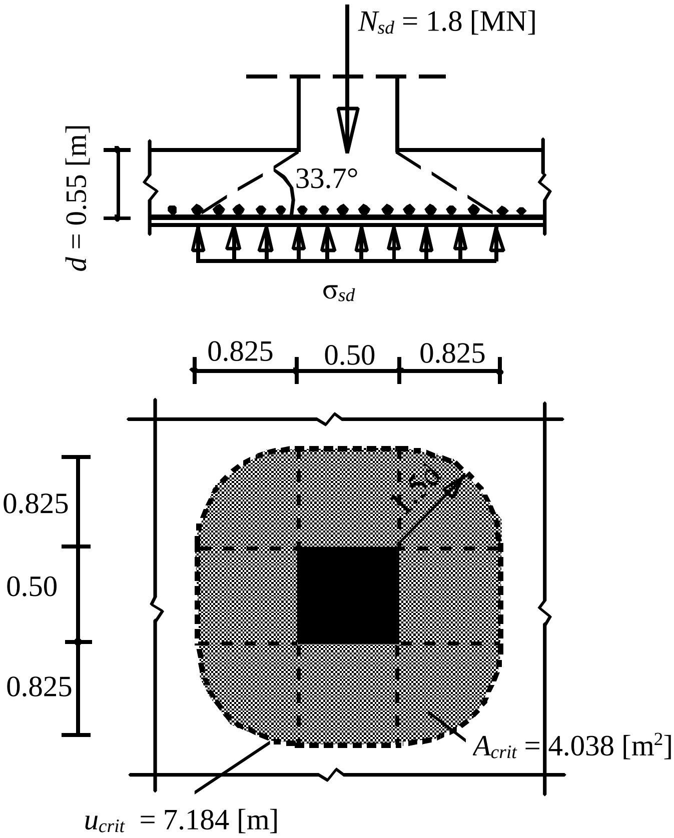

Figure 22 Critical section for punching shear according to EC 2

Geometry (Figure 22)

Effective depth of the section $d = d_x = d_y = 0.55$ [m]
Column side $c_x = c_y = 0.5$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\ d = 1.5 \times 0.55 = 0.825$ [m]
Area of critical punching shear section
$A_{crit} = c_x{}^2 + 4\ r\ c_x + \pi\ r^2 = (0.5)^2 + 4 \times 0.825 \times 0.5 + \pi\ 0.825^2 = 4.038$ [m^2]
Perimeter of critical punching shear section $u_{crit} = 4c_x + 2\ \pi\ r = 4 \times 0.5 + 2\ \pi\ 0.825 = 7.184$ [m]
Width of punching section $b_x = b_y = c_x + 2\ r = 0.5 + 2 \times 0.825 = 2.15$ [m]
Correction factor for interior column $\beta = 1.15$
Coefficient for consideration of the slab thickness $k = 1.6 - d = 1.6 - 0.55 = 1.05$ [m] > 1.0 [m]
Steel ratio $\rho_1 = \rho_{1x} = \rho_{1y} = A_{sx}/\ (b_y\ d_x\ ) = (A_s \times 10^{-4})/\ (0.55) = 0.00018\ A_s$

Loads and stresses

Total load factor for both dead and live loads	$\gamma = 1.5$
Column load	N = 1200 [kN] = 1.2 [MN]
Factored column load	$N_{sd} = \gamma$ N = 1.5 × 1.2 = 1.8 [MN]
Factored upward soil pressure under the column	$\sigma_{Sd} = \gamma\ \sigma_o$

Main value of shear strength for concrete C 30/37 according to Table 1

τ_{Rd} = 1.2 × 0.28 = 0.336 [MN/ m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{sd} - \sigma_{Sd} A_{crit}$$

$$V_{Sd} = 1.8 - 4.038\ \sigma_{Sd} \text{ [MN]}$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{(1.8 - 4.038\sigma_{Sd})1.15}{7.184} = 0.288 - 0.646\sigma_{Sd} \text{ [MN/ m]}$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd} k(1.2 + 40\rho_1)d$$

$$\nu_{Rd1} = 0.336 \times 1.05\ (1.2 + 40 \times 0.00018\ A_s)\ 0.55$$

$$\nu_{Rd1} = 0.233 + 0.0014\ A_s \text{ [MN/ m]}$$

Table 17 shows the check for punching shear by application of different soil models where for all soil models $\nu_{Sd} < \nu_{Rd1}$. Therefore, the section is safe for punching shear.

Table 17 Check for punching shear by application of different soil models

Soil model		σ_{sd} [MN/ m^2]	A_s [cm^2/ m]	ν_{Sd} [MN/ m]	ν_{Rd1} [MN/ m]
Simple assumption model	1	0.095	30.40	0.227	0.276 > ν_{Sd}
Winkler=s model	2	0.093	30.40	0.228	0.276 > ν_{Sd}
Isotropic elastic half-space medium	5	0.063	38.00	0.247	0.286 > ν_{Sd}
Layered medium	7	0.068	34.20	0.244	0.281 > ν_{Sd}

5 Design for DIN 1045

5.1 Design for flexure moment

Material

Concrete grade	B 35
Steel grade	BSt 500
Concrete compressive strength	β_R = 23 [MN/ m^2]
Tensile yield strength of steel	β_S = 500 [MN/ m^2]

Geometry

Effective depth of the section	h = 0.55 [m]
Width of the section to be designed	b = 1.0 [m]

Determination of tension reinforcement

The design of sections is carried out for DIN 1045 in table forms. Table 18 and Table 19 show the design of section *I-I*.

The normalized design moment m_s is

$$m_s = \frac{M_s}{bh^2\left(\frac{\alpha_R \beta_R}{\gamma}\right)}$$

$$m_s = \frac{M_s}{1.0 \times 0.55^2\left(\frac{0.95 \times 23}{1.75}\right)} = 0.264765 M_s$$

The normalized steel ratio ω_M is

$$\omega_M = 1 - \sqrt{1 - 2 m_s}$$

$$\omega_M = 1 - \sqrt{1 - 2 \times 0.264765 M_s} = 1 - \sqrt{1 - 0.5259 M_s}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega_M \left(\frac{(\alpha_R \beta_R) bh}{\beta_S} \right)$$

$$A_s = \omega_M \left(\frac{(0.95 \times 23) 1.0 \times 0.55}{500} \right) = 0.024035 \omega_M \ [\mathrm{m}^2/\ \mathrm{m}]$$

$$A_s = 240.35\ \omega_M\ [\mathrm{cm}^2/\ \mathrm{m}]$$

Table 18 Required bottom reinforcement under the column A_{sc} for different soil models

Soil model		M_s [MN.m/ m]	m_s	ω_M	A_{sc} [cm^2/ m]
Simple assumption model	1	0.400	0.106	0.112	26.97
Winkler's model	2	0.399	0.106	0.112	26.91
Isotropic elastic half-space medium	5	0.504	0.134	0.144	34.59
Layered medium	7	0.492	0.130	0.140	33.64

Table 19 Required top reinforcement in the field A_{sf} for different soil models

Soil model		M_s [MN.m/ m]	m_s	ω_M	A_{sf} [cm^2/ m]
Simple assumption model	1	0.013	0.00348	0.00348	0.84
Winkler's model	2	0.015	0.00394	0.00395	0.95
Isotropic elastic half-space medium	5	-	-	-	-
Layered medium	7	-	-	-	-

Chosen reinforcement

Table 20 shows the number of steel bars under the column and in the field between columns at section *I-I* considering different soil models. The chosen diameter of steel bars is Φ = 22 [mm].

Table 20 Chosen reinforcement at section *I-I* for different soil models

Soil model		Chosen reinforcement	
		Bottom *Rft* under column A_{sc}	Top *Rft* in the field A_{sf}
Simple assumption model	1	8 Φ 22 = 30.40 [cm^2/ m]	*min* A_s
Winkler=s model	3	8 Φ 22 = 30.40 [cm^2/ m]	*min* A_s
Isotropic elastic half-space medium	5	10 Φ 22 = 38.00 [cm^2/ m]	*min* A_s
Layered medium	7	9 Φ 22 = 34.20 [cm^2/ m]	*min* A_s

5.2 Check for punching shear

The critical section for punching shear is a circle of diameter d_r = 0.902 [m] around the circumference of the column as shown in Figure 23.

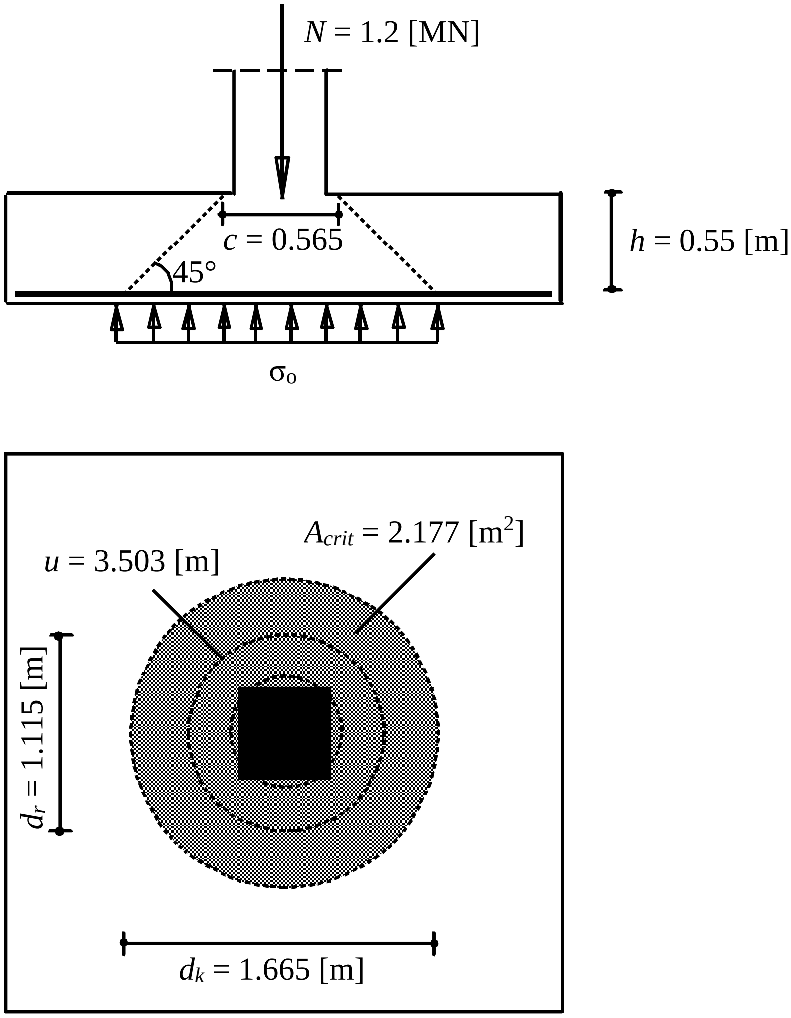

Figure 23 Critical section for punching shear according to DIN 1045

Geometry (Figure 23)

Effective depth of the section	h = 0.55 [m]
Column side	$c_x = c_x$ = 0.5 [m]
Average diameter of the column	c = 1.13 × 0.5 = 0.565 [m]
Diameter of loaded area	$d_k = 2\ h + c = 2 \times 0.55 + 0.565 = 1.665$ [m]
Diameter of critical punching shear section	$d_r = c + h = 0.565 + 0.55 = 1.115$ [m]
Area of critical punching shear section	$A_{crit} = \pi\ d_k^2/ 4 = \pi\ 1.665^2/ 4 = 2.177$ [m^2]
Perimeter of critical punching shear section	$u = \pi\ d_r = \pi\ 1.115 = 3.503$ [m]

Loads and stresses

Column load $N = 1200$ [kN] = 1.2 [MN]
Main value of shear strength for concrete B 35 according to Table 2
$\tau_{011} = 0.6$ [MN/ m^2]
Factor depending on steel grade according to Table 6 $\alpha_s = 1.4$

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o A_{crit}$$

$$Q_r = 1.2 - 2.177\sigma_o \text{ [MN]}$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{uh}$$

$$\tau_r = \frac{1.2 - 2.177\sigma_o}{3.503 \times 0.55} = 0.623 - 1.13\sigma_o \text{ [MN/ m}^2\text{]}$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sx} + A_{sy}}{2h}$$

$$\mu_g = \frac{2A_s}{2 \times 0.55 \times 100} = 0.018 A_s \text{ [\%]}$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\alpha_s \sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.4 \sqrt{0.018 A_s} = 0.245\sqrt{A_s}$$

The allowable concrete punching strength τ_{r1} [MN/ m^2] is given by

$$\tau_{r1} = \kappa_1 \tau_{011}$$

$$\tau_{r1} = 0.2454\sqrt{A_s} \times 0.6 = 0.147\sqrt{A_s} \text{ [MN/ m}^2\text{]}$$

Table 21 shows the check for punching shear by application of different soil models where for all soil models $\tau_r < \tau_{r1}$. Therefore, the section is safe for punching shear.

Table 21 Check for punching shear by application of different soil models

Soil model		σ_o [MN/ m^2]	A_s [cm^2/ m]	τ_r [MN/ m^2]	τ_{r1} [MN/ m^2]
Simple assumption model	1	0.063	30.40	0.552	0.811 > τ_r
Winkler=s model	2	0.062	30.40	0.553	0.811 > τ_r
Isotropic elastic half-space medium	5	0.042	38.00	0.576	0.906 > τ_r
Layered medium	7	0.045	34.20	0.572	0.860 > τ_r

6 Comparison between the design according to DIN 1045 and EC 2

Table 22 shows the comparison between the design of the raft according to DIN 1045 and EC 2 by application of different soil models. The comparison is considered only for bottom reinforcement under the column.

It can be concluded from the comparison that if the raft is designed according to EC 2 using a load factor of γ = 1.5 and DIN 1045, the required reinforcement obtained from EC 2 will be nearly the same as that obtained from DIN 1045. Finally, it can be concluded also from Table 16 and Table 20 that the chosen reinforcement for both EC 2 and DIN 1045 is identical.

Table 22 Comparison between the design according to DIN 1045 and EC 2

Soil model		A_s [cm²/ m] according to		Difference ΔA_s [%]
		DIN 1045	EC 2	
Simple assumption model	1	26.97	26.76	0.78
Winkler=s model	2	26.91	26.70	0.78
Isotropic elastic half-space medium	5	34.59	34.39	0.58
Layered medium	7	33.64	33.43	0.62