Example 3: Design of an irregular raft on irregular subsoil

1 Description of the problem

Cruz (1994) under the supervision of the author examined an irregular raft of high rise building on irregular subsoil by *ELPLA*. He carried out the examination to show the difference between the design of rafts according to national code (German code) and Eurocode. Here, *Kany*/ *El Gendy* (1995) have chosen the same example with some modifications. The accurate method of interpolation is used instead of subareas method to obtain the three-dimensional flexibility coefficient and modulus of subgrade reaction for Continuum and *Winkler's* models, respectively.

To carry out the comparison between the different design codes and soil models, three different soil models are used to analyze the raft. In this example, three mathematical calculation methods are chosen to represent the three soil models: Simple assumption, *Winkler's* and Continuum models as shown in Table 23.

Table 23 Calculation methods and soil models

Method No.	Calculation method	Soil model
1	Linear contact pressure method	Simple assumption model
3	Variable modulus of subgrade reaction method	*Winkler's* model
6	Modulus of compressibility method	Continuum model

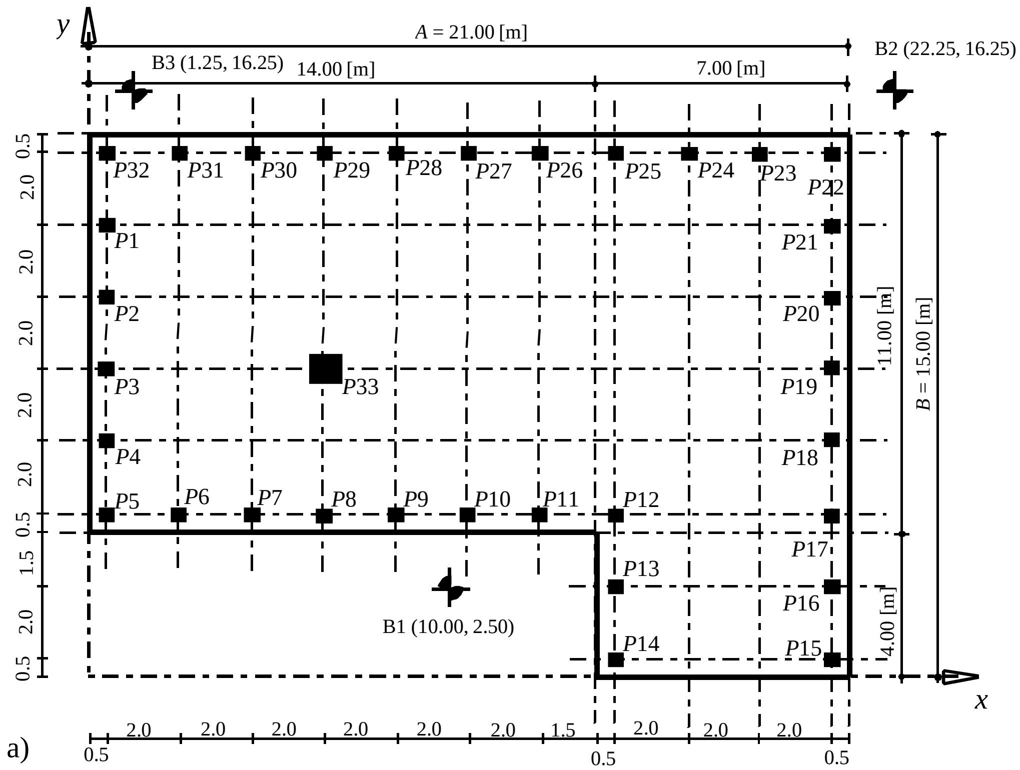
Figure 24 shows the plan of the raft, column loads, dimensions, mesh with section through the raft and subsoil. The following text gives a description of the design properties and parameters.

2 Properties of raft material and section

2.1 Material properties

Young's modulus of concrete	E_b = 34 000 000 [kN/ m^2]
Poisson's ratioof concrete	ν_b = 0.20 [-]
Unit weight of concrete	γ_b = 0 [kN/ m^3]

Unit weight of concrete is chosen γ_b = 0.0 to neglect the self-weight of the raft.



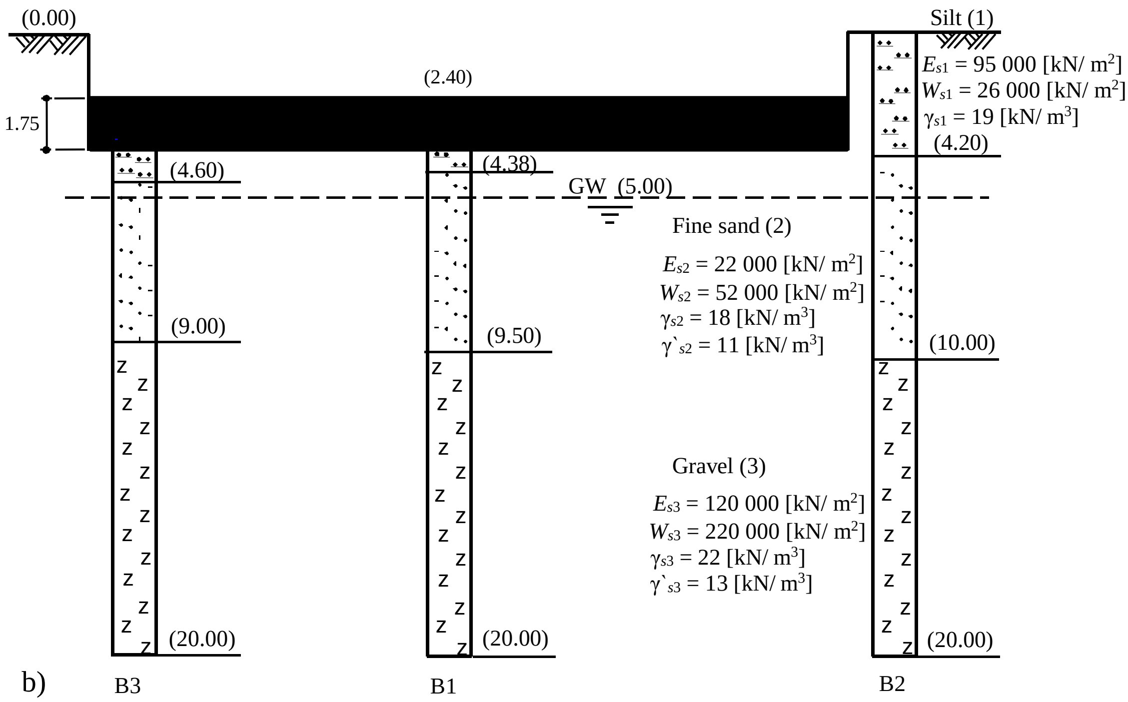


Figure 24 a) Plan of the raft with column loads, dimensions and mesh
b) Section through the raft and subsoil

2.2 Section properties

To carry out the comparison of the different codes and soil models, the raft thickness is chosen t = 1.75 [m] for all soil models and design codes. The raft section has the following parameters:

Width of the section to be designed	b = 1.0	[m]
Section thickness	t = 1.75	[m]
Concrete cover + 1/2 bar diameter	c = 5	[cm]
Effective depth of the section	$d = t - c$ = 1.70	[m]
Steel bar diameter	Φ = 25	[mm]
Minimum area of steel	$A_s\ min$ = 6 Φ 25 = 29.5 [cm^2/ m]	

3 Soil properties

Three boring logs characterize the soil under the raft. Each boring has three layers with different materials as shown in Table 24 and Figure 24. *Poisson's* ratio is constant for all the soil layers. The effect of reloading is taken into account. The general soil parameters are:

Poisson's ratio of the soil layers	ν_s = 0.25
Settlement reduction factor for sand according to DIN 4019	α = 0.66
Level of foundation depth underground surface	d_f = 4.15 [m]

Table 24 Soil properties

Layer No.	Type of soil	Depth of layer under the ground surface z [m]	Modulus of compressibility of the soil for		Unit weight above ground water γ_s [kN/ m^3]	Unit weight under ground water γ'_s [kN/ m^3]
			Loading E_s [kN/ m^2]	Reloading W_s [kN/ m^2]		
1	Silt	4.38/4.2/4.6	9 500	26 000	19	-
2	Fine sand	9.5/10.0/9.0	22 000	52 000	18	11
3	Gravel	20.0	120 000	220 000	22	13

4 Loads on the raft

The raft carries 33 column loads as shown in Figure 24. The ratio of dead to live loads N_{gk}/ N_{qk} by the analysis is 70 [%]/ 30 [%]. Table 25 shows the raft loads according to DIN 1045 and EC 2. To obtain the results according to EC 2, the analysis of the raft may be carried out once for both codes due to the given loads. Then the results are multiplied by a global factor of safety γ = 1.395, which may be obtained through the following relation

$$N_{sd} = N_{gk} + N_{qk} = \gamma_g \times G_k + \gamma_q \times Q_k = 1.35\ (0.7 \times P) + 1.5\ (0.3 \times P) = 1.395\ P$$

where:

N_{sd} Design value of action
P Given column load
γ_g Partial factor for dead action, $\gamma_g = 1.35$
γ_q Partial factor for live action, $\gamma_q = 1.5$
G_k Given dead load, $N_{gk} = 0.7 \times P$
Q_k Given live load, $N_{qk} = 0.3 \times P$
N_{gk} Factored dead load, $N_{gk} = 0.7 \times P$
N_{qk} Factored live load, $N_{qk} = 0.3 \times P$

Table 25 Loads on the raft

Column No.	Given column load N [kN]	Dead load 70 [%] N G_k [kN]	Live load 30 [%] N Q_k [kN]	$N_{gk} = \gamma_g\, G_k$ ($\gamma_g = 1.35$) [kN]	$N_{qk} = \gamma_q\, Q_k$ ($\gamma_q = 1.50$) [kN]	$N_{sd} = N_{gk} + N_{qk}$ [kN]
P22, P32	980	686	294	926	441	1367
P23 to P31	1350	945	405	1276	608	1883
P16 to P21	1380	966	414	1304	621	1925
P14, P15	1150	805	345	1087	518	1604
P13	1000	700	300	945	450	1395
P1 to P4, P12	1250	875	375	1181	563	1744
P6 to P11	1200	840	360	1134	540	1674
P5	990	693	297	936	446	1381
P33	10490	7343	3147	9913	4720	14634

5 Analysis of the raft

The raft is subdivided into 106 elements. Then, the analysis of the raft according to both the two codes DIN 1045 and EC 2 is carried out by *ELPLA*. The system of linear equations of the Continuum model is solved by iteration (method 6). The maximum difference between the soil settlement s [cm] and the raft deflection w [cm] is considered as an accuracy number. In this example, the accuracy is chosen $\varepsilon = 0.001$ [cm]. Because the raft is subdivided into a mesh of coarse finite elements, representing the column load by a loaded area instead of point load is not necessary.

Determination of main modulus of subgrade reaction k_{sm} for the three boring logs

Main modulus k_{sm} for each boring log should be determined. Each modulus is corresponding to one of the soil boring logs and is calculated from the elastic materials of that boring. The main moduli of subgrade reactions k_{sm} for the three boring logs are:

k_{sm1} = 12936 [kN/ m^3]
k_{sm2} = 12799 [kN/ m^3]
k_{sm3} = 13109 [kN/ m^3]

Determination of variable modulus of subgrade reaction $k_{s.i}$

According to *Kany*/ *El Gendy* (1995), the raft area is divided into three region types as shown in Figure 25.

Type I
This region is a triangular region. The three boring logs B1 to B3 confine that region. The modulus $k_{s.i}$ for a node inside the triangular region can be determined by interpolation through the values of k_{sm} for the three boring logs

Type II
One or more sides of the raft and two boring logs confine this region (regions of B1 and B2, B1 and B3). Assuming a linear interpolation between the values of k_{sm} for the two boring logs can obtain the modulus $k_{s.i}$ for a node i inside this region

Type III
One or more sides of the raft and one boring confine this region. The modulus $k_{s.i}$ for a node inside this region is equal to the modulus of that boring. For the considered raft, the regions of type III are outside the raft area

Figure 26 shows the calculated variable modulus of subgrade reaction $k_{s.i}$ according to the interpolation method. In a similar way to the previous solution for *Winkler's* model, the three-dimensional coefficient of flexibility can be determined for Continuum model.

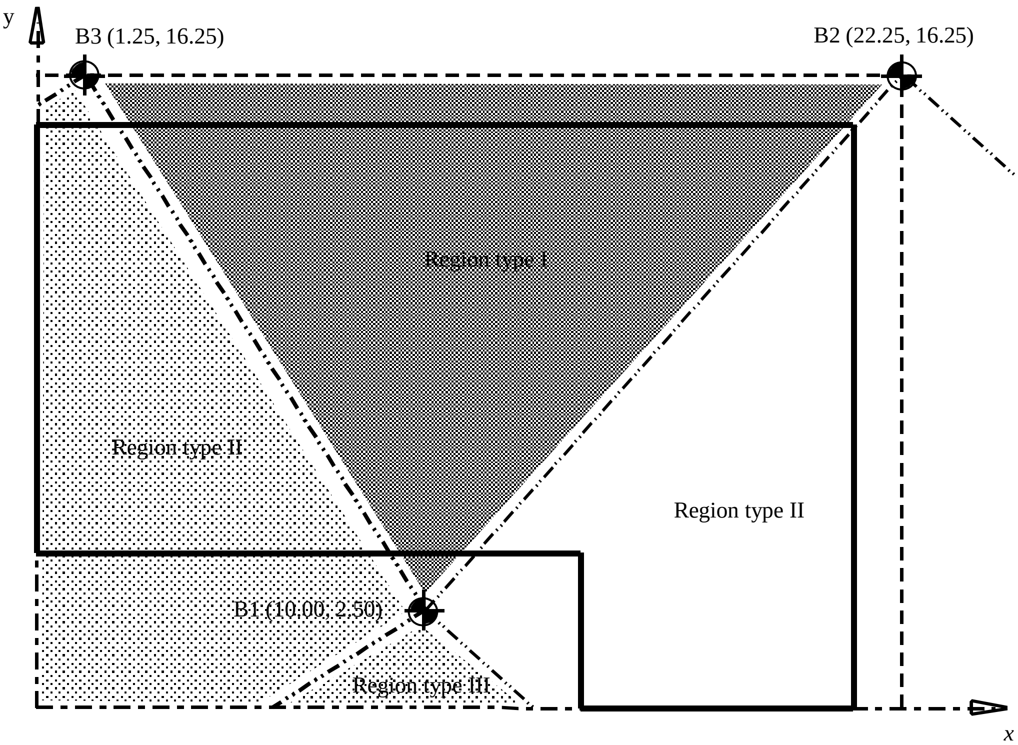


Figure 25 Boring locations and region types I, II and III

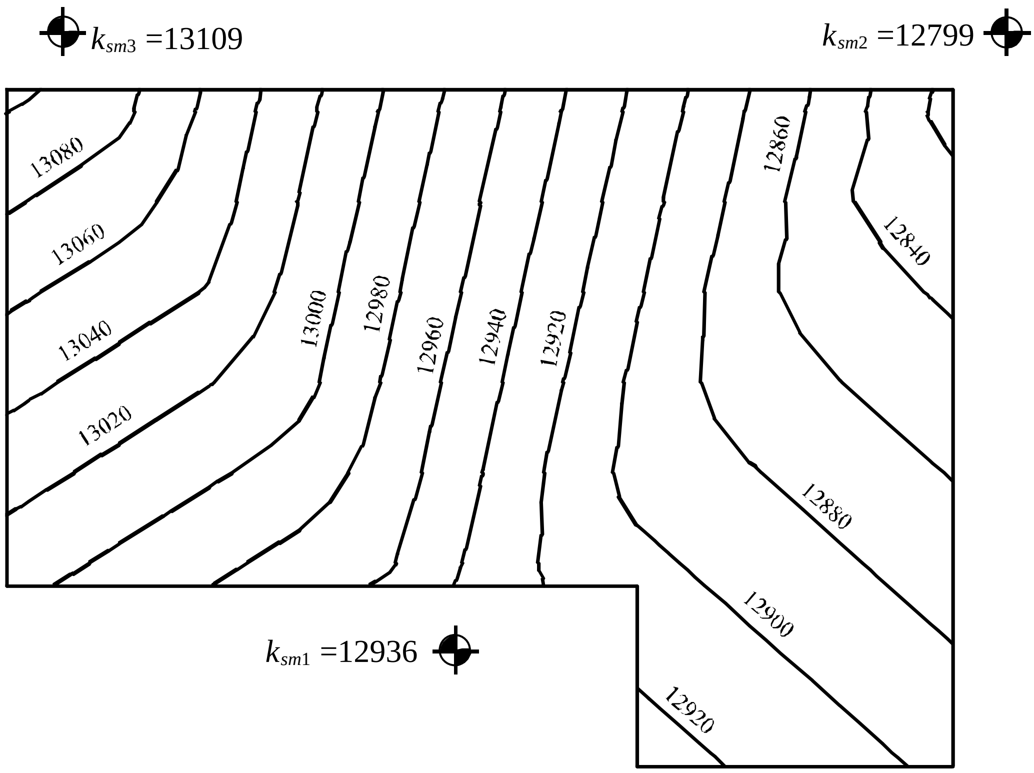


Figure 26 Contour lines for variable modulus of subgrade reaction k_s [kN/ m^3]

Definition of the critical sections

Two critical sections in *x*- and *y*-directions passing through the heavy loaded column P33 are considered as shown in Figure 27. In this example, the design is carried out only for the critical sections *x*-*x* and *y*-*y* in detail. Figure 28 to 29 and Table 26 show the contact pressure under the column σ_o, field moment m_f and the column moment m_c at the critical sections *x*-*x* and *y*-*y* by application of different soil models. For the codes DIN 1045 and EC 2, the sections are designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained. Finally, a comparison of the results of the two codes and soil models is presented.

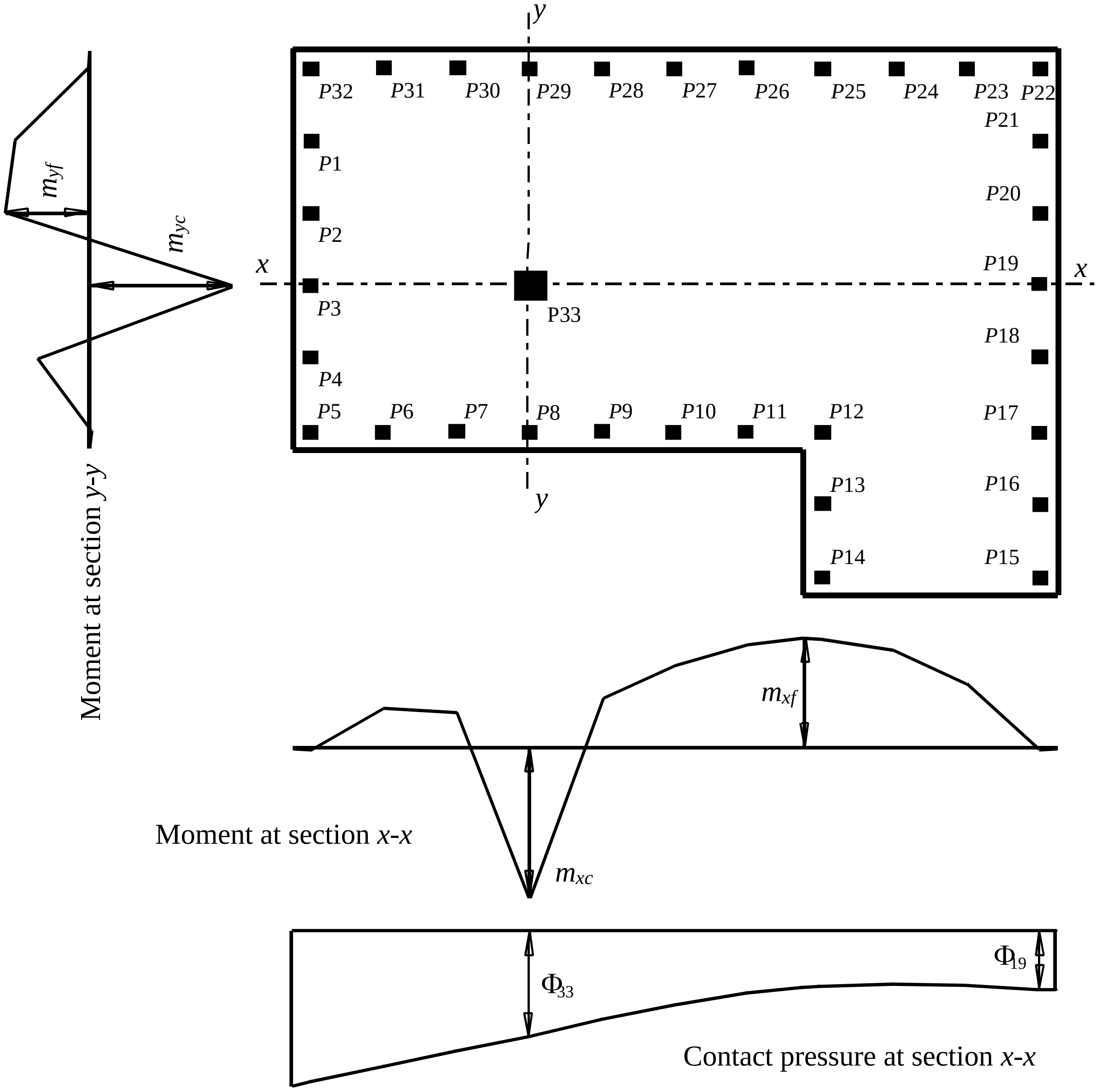


Figure 27 Definition of critical sections in *x*- and *y*-directions

Table 26 Contact pressure σ_o under the column, field moment m_f and column moment m_c at the critical sections *x-x* and *y-y* by application of different soil models

Soil model		Contact pressure [kN/ m^2]		Column moment [kN.m/ m]		Field moment [kN.m/ m]	
		σ_{33}	σ_{19}	m_{xc}	m_{yc}	m_{xf}	m_{yf}
Simple assumption model	1	221	145	1444	1424	-2182	-1137
Winkler's model	3	217	164	1827	1527	-1728	-1026
Continuum model	6	181	159	2320	1866	-1292	-694

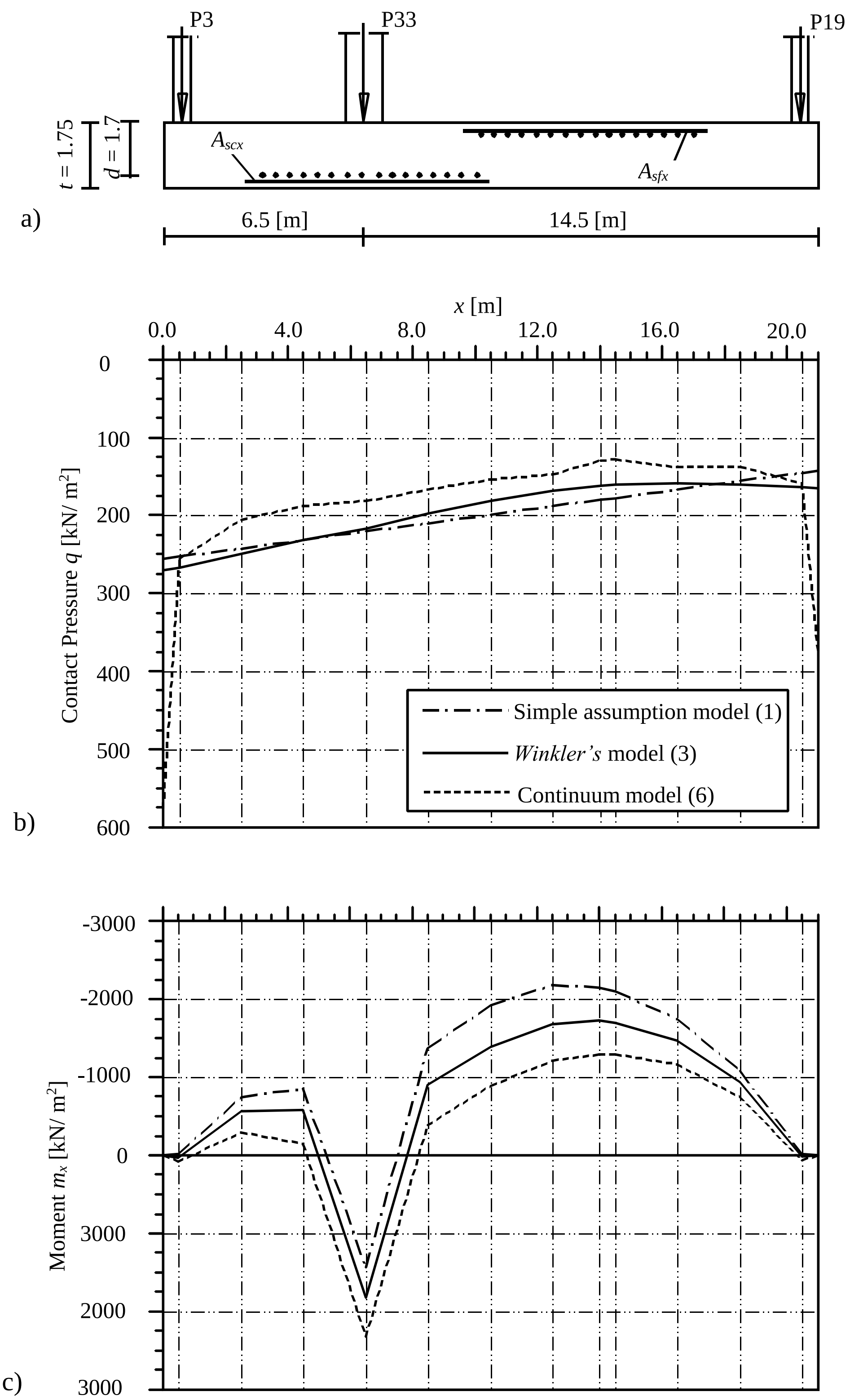


Figure 28 a) Section *x*-*x* through the raft
b) Moment m_x [kN.m/ m] at section *x*-*x*
c) Contact pressure q [kN/ m^2] at section *x*-*x*

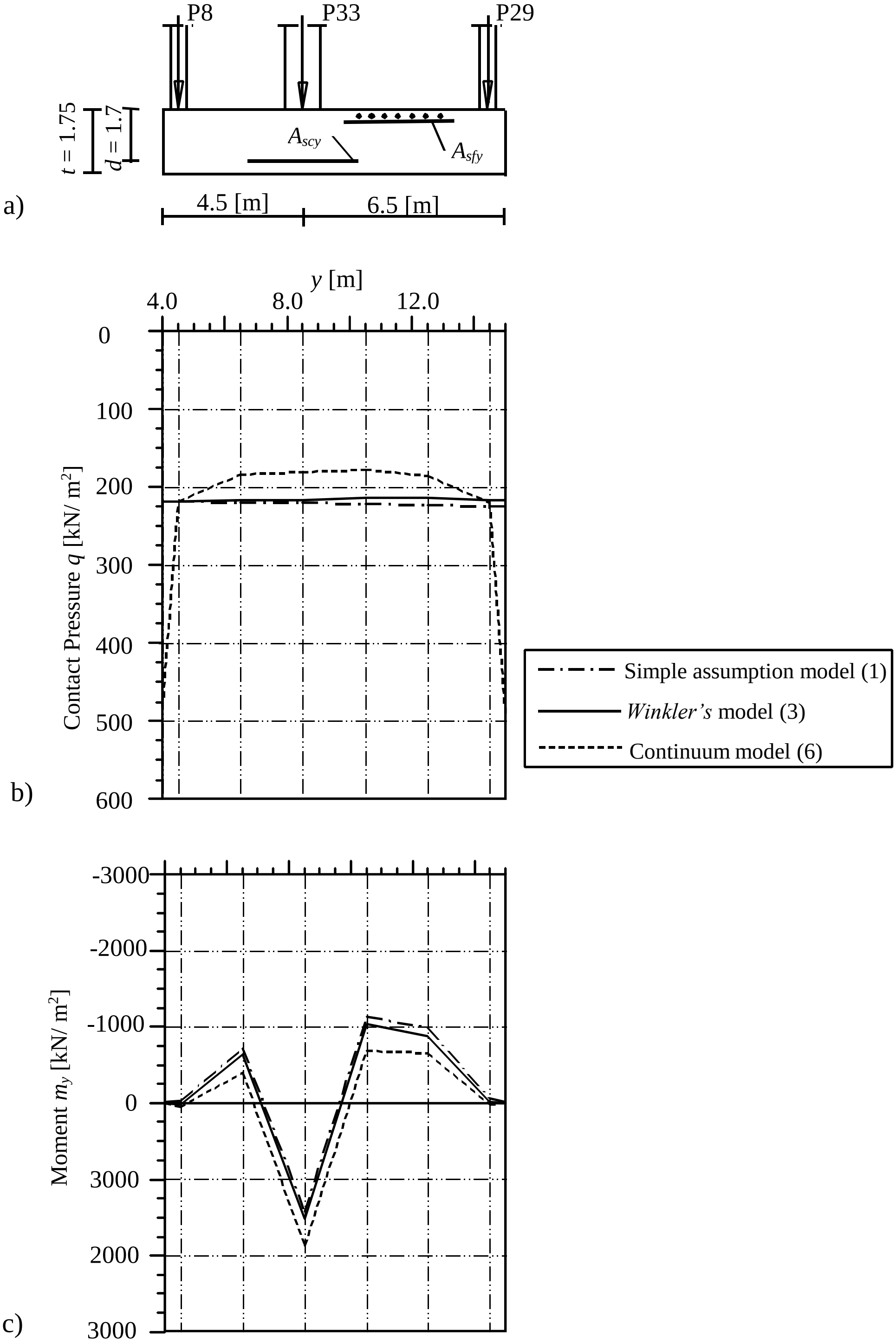


Figure 29 a) Section *y*-*y* through the raft
b) Moment m_y [kN.m/ m] at section *y*-*y*
c) Contact pressure *q* [kN/ m^2] at section *y*-*y*

6 Design for EC 2

6.1 Design for flexure moment

Material

Concrete grade	C 30/37
Steel grade	BSt 500
Characteristic compressive cylinder strength of concrete	f_{ck} = 30 [MN/ m^2]
Characteristic tensile yield strength of reinforcement	$f_{yk} = f_y$ = 500 [MN/ m^2]
Partial safety factor for concrete strength	$\gamma_c = 1.5$
Design concrete compressive strength	$f_{cd} = f_{ck}/\gamma_c$ = 30/1.5 = 20 [MN/ m^2]
Partial safety factor for steel strength	$\gamma_s = 1.15$
Design tensile yield strength of reinforcing steel	$f_{yd} = f_{yk}/\gamma_s$ = 500/1.15 = 435 [MN/ m^2]

Factored moment

Total load factor for both dead and live loads	$\gamma = 1.395$
Factored column moment	$M_{sd} = \gamma m_c$
Factored field moment	$M_{sd} = \gamma m_f$

Geometry

Effective depth of the section	d = 1.7 [m]
Width of the section to be designed	b = 1.0 [m]

Determination of tension reinforcement

The design of sections is carried out for EC 2 in table forms. Table 27 to Table 30 show the design of sections *x*-*x* and *y*-*y*.

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 1.7^2 (0.85 \times 20)} = 0.0204 M_{sd}$$

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.0204 M_{sd}} = 1 - \sqrt{1 - 0.0408 M_{sd}}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega\left(\frac{(0.85 f_{cd})bd}{f_{yd}}\right)$$

$$A_s = \omega\left(\frac{(0.85 \times 20) \times 1.0 \times 1.7}{435}\right) = 0.0664368\omega \ [\mathrm{m}^2/\mathrm{m}]$$

$$A_s = 664.368\omega \ [\mathrm{cm}^2/\mathrm{m}]$$

Table 27 Required bottom reinforcement under the column A_{sxc} for different soil models (section *x*-*x*)

Soil model		M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{sxc} [cm²/ m]
Simple assumption model	1	2.014	0.041	0.042	27.89
Winkler's model	3	2.549	0.052	0.053	35.50
Continuum model	6	3.236	0.066	0.068	45.40

Table 28 Required top reinforcement in the field A_{sxf} for different soil models (section *x*-*x*)

Soil model		M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{sxf} [cm²/ m]
Simple assumption model	1	3.044	0.062	0.064	42.62
Winkler's model	3	2.411	0.049	0.050	33.52
Continuum model	6	1.802	0.037	0.038	24.89

Table 29 Required bottom reinforcement under the column A_{syc} for different soil models (section *y*-*y*)

Soil model		M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{syc} [cm^2/ m]
Simple assumption model	1	1.987	0.041	0.041	27.49
Winkler's model	3	2.130	0.044	0.044	29.53
Continuum model	6	2.603	0.053	0.055	36.27

Table 30 Required top reinforcement in the field A_{syf} for different soil models (section *y*-*y*)

Soil model		M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{syf} [cm^2/ m]
Simple assumption model	1	1.586	0.032	0.033	21.86
Winkler's model	3	1.431	0.029	0.030	19.69
Continuum model	6	0.968	0.020	0.020	13.25

Chosen reinforcement

Table 31 and Table 32 show the number of steel bars under the column and in the field between columns at sections *x-x* and *y-y* considering different soil models. The chosen diameter of steel bars is Φ = 25 [mm].

Table 31 Chosen reinforcement at section *x-x* for different soil models

Soil model		Chosen reinforcement	
		Bottom *Rft* under the column A_{sxc}	Top *Rft* in the field A_{sxf}
Simple assumption model	1	*min* A_s = 29.50 [cm²/ m]	9 Φ 25 = 44.20 [cm²/ m]
Winkler's model	3	8 Φ 25 = 39.30 [cm²/ m]	7 Φ 25 = 34.40 [cm²/ m]
Continuum model	6	10 Φ 25 = 49.10 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]

Table 32 Chosen reinforcement at section *y-y* for different soil models

Soil model		Chosen reinforcement	
		Bottom *Rft* under the column A_{syc}	Top *Rft* in the field A_{syf}
Simple assumption model	1	*min* A_s = 29.50 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]
Winkler's model	3	*min* A_s = 29.50 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]
Continuum model	6	8 Φ 25 = 39.30 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]

6.2 Check for punching shear

6.2.1 Interior column (column P33)

The critical punching shear section for interior columns is considered at column P33. The column dimensions are chosen to be 90/ 90 [cm]. The critical section for punching shear is at a distance *r* = 2.55 [m] around the circumference of the column as shown in Figure 30.

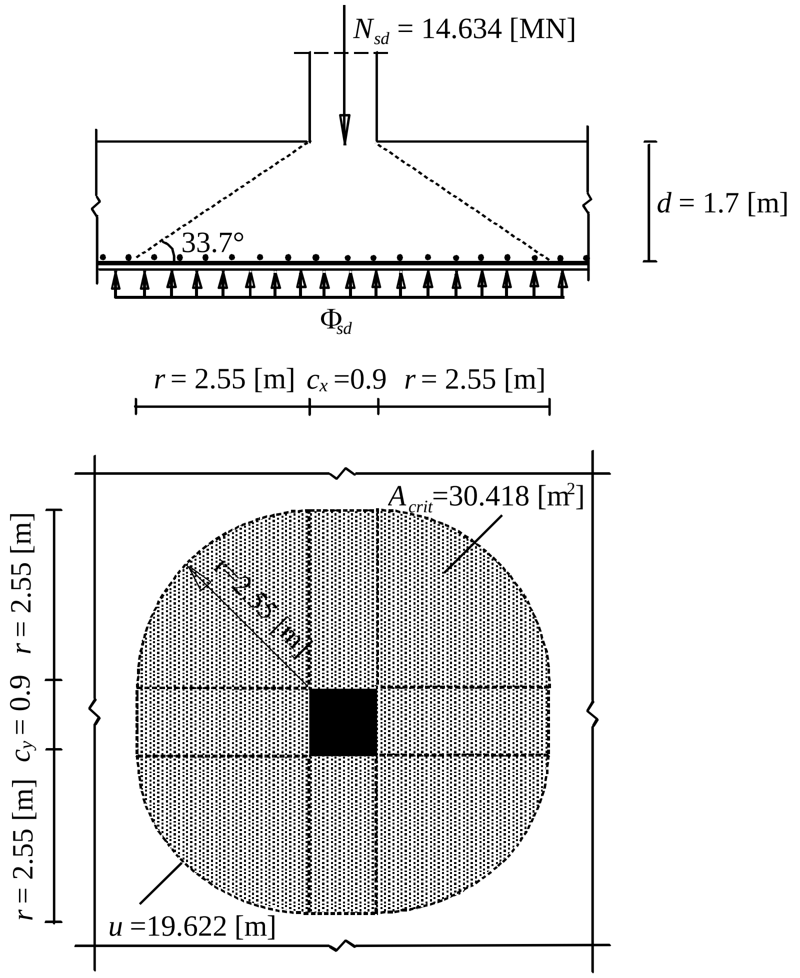


Figure 30 Critical section for punching shear according to EC 2

Geometry (Figure 30)

Effective depth of the section $d = d_x = d_y = 1.70$ [m]
Column side $c_x = c_y = 0.9$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\ d = 1.5 \times 1.70 = 2.55$ [m]
Area of critical punching shear section
$A_{crit} = c_x^{\ 2} + 4\ r\ c_x + \pi\ r^2 = (0.9)^2 + 4 \times 2.55 \times 0.9 + \pi\ 2.55^2 = 30.418$ [m^2]
Perimeter of critical punching shear section $u_{crit} = 4\ c_x + 2\ \pi\ r = 4 \times 0.9 + 2\ \pi\ 2.55 = 19.622$ [m]
Width of punching section $b_x = b_y = c_x + 2\ r = 0.9 + 2 \times 2.55 = 6.0$ [m]
Correction factor for interior column $\beta = 1.15$
Coefficient for consideration of the slab thickness $k = 1.6 - d = 1.6 - 1.70 = -0.1 < 1.0$ [m]
k is taken 1.0 [m]
Steel ratio $\rho_{1x} = A_{sx}/ (b_y\ d_x) = (A_{sxc} \times 10^{-4})/ (1.70) = 0.0000588\ A_{sxc}$
Steel ratio $\rho_{1y} = A_{sy}/ (b_x\ d_y) = (A_{syc} \times 10^{-4})/ (1.70) = 0.0000588\ A_{syc}$
Steel ratio $\rho_1 = (\rho_{1x}\ \rho_{1y})^{1/2} = 0.0000588\ (A_{sxc}\ A_{syc})^{1/2}$

Loads and stresses

Total load factor for both dead and live loads	$\gamma = 1.395$
Column load	$N = 10490$ [kN] = 10.49 [MN]
Factored column load	$N_{sd} = \gamma\ N = 1.395 \times 10.49 = 14.634$ [MN]
Factored upward soil pressure under the column	$\sigma_{sd} = \gamma\ \sigma_o$
Main value of shear strength for concrete C 30/37 according to Table 1	
	$\tau_{Rd} = 1.2 \times 0.28 = 0.336$ [MN/m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{sd} - \sigma_{Sd}\ A_{crit}$$

$$V_{Sd} = 14.634 - 30.418\ \sigma_{Sd}\ \text{[MN]}$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{(14.634 - 30.418\sigma_{Sd})1.15}{19.622} = 0.858 - 1.783\sigma_{Sd}\ \text{[MN/ m]}$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd} k (1.2 + 40\rho_1) d$$

$$\nu_{Rd1} = 0.336 \times 1.0 (1.2 + 40 \times 0.0000588 \sqrt{A_{sxc} A_{syc}}) 1.7$$

$$\nu_{Rd1} = 0.68544 + 0.001344 \sqrt{A_{sxc} A_{syc}} \text{ [MN/ m]}$$

Table 33 shows the check of punching shear for the interior column P33 by application of different soil models where for all soil models $\nu_{sd} < \nu_{Rd1}$. Therefore, the section is safe for punching shear.

Table 33 Check of punching shear for the interior column P33 by application of different soil models

Soil model		σ_{sd} [MN/ m^2]	$(A_{sxc}\, A_{sxc})^{1/2}$ [cm^2/ m]	ν_{sd} [MN/ m]	ν_{Rd1} [MN/ m]
Simple assumption model	1	0.308	29.5	0.309	0.725 > ν_{sd}
Winkler's model	3	0.303	34.05	0.318	0.731 > ν_{sd}
Continuum model	6	0.253	43.93	0.407	0.745 > ν_{sd}

6.2.2 Exterior column (column P19)

The critical punching shear section for exterior column is considered at column P19 (Figure 31).

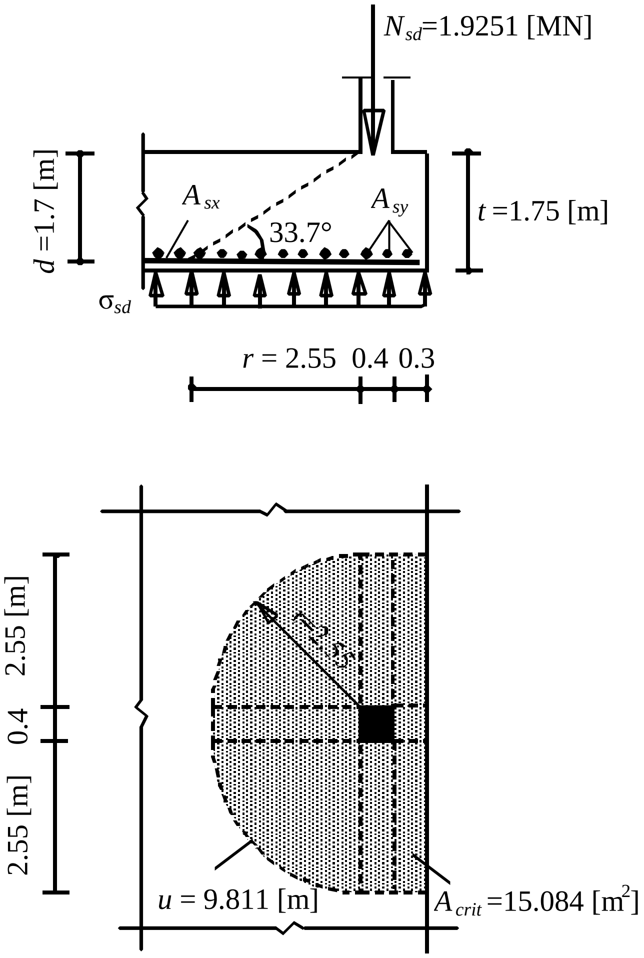


Figure 31 Critical section for punching shear according to EC 2

Geometry (Figure 31)

Effective depth of the section $d = d_x = d_y = 1.7$ [m]
Column side $c_x = c_y = 0.4$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\,d = 1.5 \times 1.7 = 2.55$ [m]
Area of critical punching shear section
$A_{crit} = c_x\, c_y + 2\, r\, c_x + r\, c_y + 0.3\,(2r + c_y) + 0.5\pi\, r^2$
$= (0.4)^2 + 3 \times 2.55 \times 0.4 + 0.3\,(2 \times 2.55 + 0.4) + 0.5\pi\, 2.55^2$
$A_{crit} = 15.084$ [m^2]
Perimeter of critical punching shear section
$u_{crit} = 2c_x + c_y + 2 \times 0.3 + \pi\, r = 3 \times 0.4 + 2 \times 0.3 + \pi\, 2.55 = 9.811$ [m]
Width of punching section $b_x = 0.3 + c_x + r = 0.3 + 0.4 + 2.55 = 3.25$ [m]
Width of punching section $b_y = c_y + 2\, r = 0.4 + 2 \times 2.55 = 5.5$ [m]
Correction factor for edge column $\beta = 1.4$

Coefficient for consideration of the slab thickness k = 1.6 - d = 1.6 - 1.7 = - 0.1 < 1.0 [m]
k is taken 1.0 [m]
Steel ratio $\rho_1 = \rho_{1x} = \rho_{1y} = (min\ A_s \times 10^{-4}) / (1.7) = 0.00174$

Loads and stresses

Total load factor for both dead and live loads	$\gamma = 1.395$
Column load	N = 1380 [kN] = 1.38 [MN]
Factored column load	$N_{sd} = \gamma\ N = 1.395 \times 1.38 = 1.9251$ [MN]
Factored upward soil pressure under the column	$\sigma_{sd} = \gamma\ \sigma_o$

Main value of shear strength for concrete C 30/37 according to Table 1
$\tau_{Rd} = 1.2 \times 0.28 = 0.336$ [MN/ m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{sd} - \sigma_{Sd}\ A_{crit}$$

$$V_{Sd} = 1.9251 - 15.084\ \sigma_{Sd} \text{ [MN]}$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{(1.9251 - 15.084\sigma_{Sd})1.40}{9.811} = 0.275 - 2.152\sigma_{Sd} \text{ [MN/ m]}$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd}\, k(1.2 + 40\rho_1)d$$

$$\nu_{Rd1} = 0.336 \times 1.0\ (1.2 + 40 \times 0.00174)\ 1.7$$

$$\nu_{Rd1} = 0.725 \text{ [MN/ m]}$$

Table 34 shows the check of punching shear for the exterior column P19 by application of different soil models where for all soil models $\nu_{Sd} < \nu_{Sd1}$. Therefore, the section is safe for punching shear.

Table 34 Check of punching shear for the exterior column P19 by application of different soil models

Soil model		σ_{sd} [MN/ m^2]	$(A_{sxc} \times A_{sxc})^{1/2}$ [cm^2/ m]	ν_{sd} [MN/ m]	ν_{Rd1} [MN/ m]
Simple assumption model	1	0.202	29.5	0.160	0.725 > ν_{sd}
Winkler's model	3	0.229	29.5	0.217	0.725 > ν_{sd}
Continuum model	6	0.222	29.5	0.202	0.725 > ν_{sd}

7 Design for DIN 1045

7.1 Design for flexure moment

Material

Concrete grade	B 35
Steel grade	BSt 500
Concrete compressive strength	β_R = 23 [MN/ m^2]
Tensile yield strength of steel	β_S = 500 [MN/ m^2]

Geometry

Effective depth of the section	h = 1.7 [m]
Width of the section to be designed	b = 1.0 [m]

Determination of tension reinforcement

The design of sections is carried out for DIN 1045 in table forms. Table 35 to Table 38 show the design of sections *x*-*x* and *y*-*y*.

The normalized design moment m_s is

$$m_s = \frac{M_s}{bh^2\left(\frac{\alpha_R \beta_R}{\gamma}\right)}$$

$$m_s = \frac{M_s}{1.0 \times 1.7^2\left(\frac{0.95 \times 23}{1.75}\right)} = 0.027713 M_s$$

The normalized steel ratio ω_M is

$$\omega_M = 1 - \sqrt{1 - 2m_s}$$

$$\omega_M = 1 - \sqrt{1 - 2 \times 0.027713 M_s} = 1 - \sqrt{1 - 0.0554 M_s}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega_M \left(\frac{(\alpha_R \beta_R) b h}{\beta_S} \right)$$

$$A_s = \omega_M \left(\frac{(0.95 \times 23) 1.0 \times 1.7}{500} \right) = 0.07429 \omega_M \ [\text{m}^2/\text{ m}]$$

$$A_s = 742.90\ \omega_M\ [\text{cm}^2/\text{ m}]$$

Table 35 Required bottom reinforcement under the column A_{sxc} for different soil models (section *x-x*)

Soil model		M_s [MN.m/ m]	m_s	ω_M	A_{sxc} [cm^2/ m]
Simple assumption model	1	1.444	0.040	0.041	30.35
Winkler's model	3	1.827	0.051	0.052	38.62
Continuum model	6	2.320	0.064	0.067	49.41

Table 36 Required top reinforcement in the field A_{sxf} for different soil models (section *x-x*)

Soil model		M_s [MN.m/ m]	m_s	ω_M	A_{sxf} [cm^2/ m]
Simple assumption model	1	2.182	0.061	0.062	46.37
Winkler's model	3	1.728	0.048	0.049	36.47
Continuum model	6	1.292	0.036	0.037	27.09

Table 37 Required bottom reinforcement under the column A_{syc} for different soil models (section *y*-*y*)

Soil model		M_s [MN.m/ m]	m_s	ω_M	A_{syc} [cm^2/ m]
Simple assumption model	1	1.424	0.040	0.040	29.92
Winkler's model	3	1.527	0.042	0.043	32.13
Continuum model	6	1.866	0.052	0.053	39.47

Table 38 Required top reinforcement in the field A_{syf} for different soil models (section *y*-*y*)

Soil model		M_s [MN.m/ m]	m_s	ω_M	A_{syf} [cm^2/ m]
Simple assumption model	1	1.137	0.032	0.032	23.79
Winkler's model	3	1.026	0.028	0.029	21.43
Continuum model	6	0.694	0.019	0.019	14.43

Chosen reinforcement

Table 39 and Table 40 show the number of steel bars under the column and in the field between columns at sections *x*-*x* and *y*-*y* considering different soil models. The chosen diameter of steel bars is Φ = 25 [mm].

Table 39 Chosen reinforcement at section *x*-*x* for different soil models

Soil model		Chosen reinforcement	
		Bottom *Rft* under the column A_{sxc}	Top *Rft* in the field A_{sxf}
Simple assumption model	1	7 Φ 25 = 34.40 [cm²/ m]	10 Φ 25 = 49.10 [cm²/ m]
Winkler's model	3	8 Φ 25 = 39.30 [cm²/ m]	8 Φ 25 = 39.30 [cm²/ m]
Continuum model	6	11 Φ 25 = 54.01 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]

Table 40 Chosen reinforcement at section *y*-*y* for different soil models

Soil model		Chosen reinforcement	
		Bottom *Rft* under the column A_{syc}	Top *Rft* in the field A_{syf}
Simple assumption model	1	*min* A_s = 29.50 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]
Winkler's model	3	7 Φ 25 = 34.40 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]
Continuum model	6	9 Φ 25 = 44.20 [cm²/ m]	*min* A_s = 29.50 [cm²/ m]

7.2 Check for punching shear

7.2.1 Interior column (column P33)

The critical punching shear section for interior column is considered at column P33. The column dimensions are chosen to be 90/ 90 [cm]. The critical section for punching shear is a circle of diameter d_r = 2.717 [m] around the circumference of the column as shown in Figure 32.

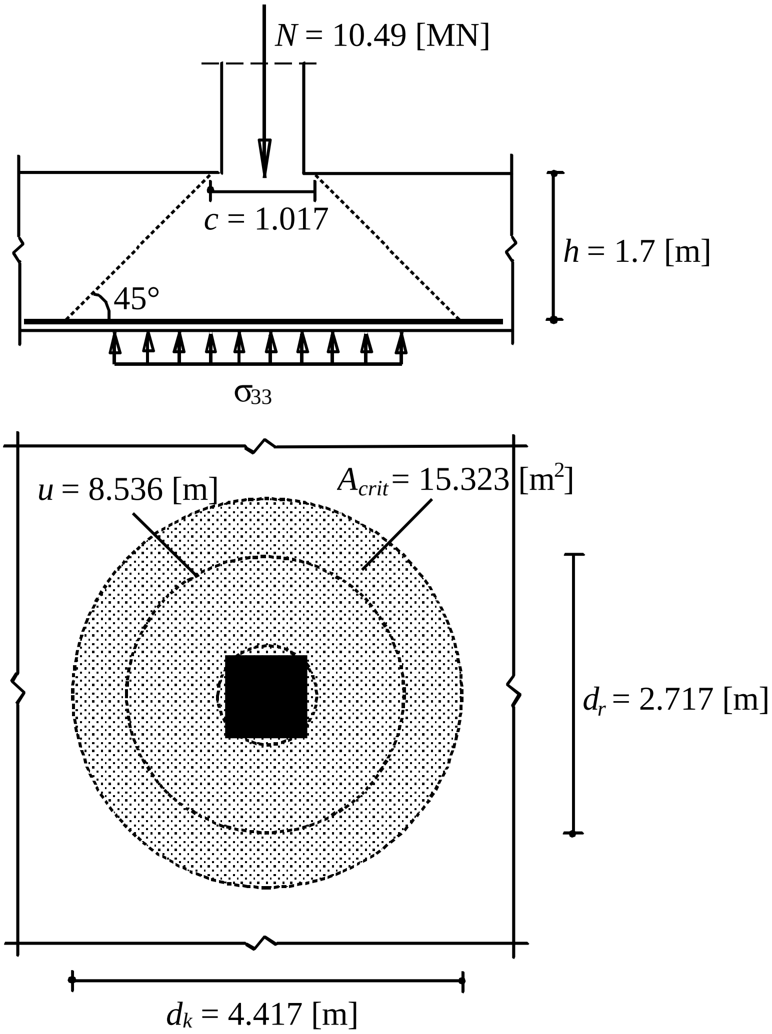


Figure 32 Critical section for punching shear according to DIN 1045

Geometry (Figure 32)

Effective depth of the section	h = 1.7 [m]
Column side	$c_x = c_y$ = 0.9 [m]
Average diameter of the column	c = 1.13 (0.9 × 0.9)$^{1/2}$ = 1.017 [m]
Diameter of loaded area	d_k = 2 h + c = 2 × 1.7 + 1.017 = 4.417 [m]
Diameter of critical punching shear section	d = c + h = 1.017 + 1.7 = 2.717 [m]
Area of critical punching shear section	$A_{crit} = \pi\, d_k^2/4 = \pi\, 4.417^2/4$ = 15.323 [m^2]
Perimeter of critical punching shear section	$u = \pi\, d_r = \pi$ 2.717 = 8.536 [m]

Loads and stresses

Column load	N = 10490 [kN] = 10.49 [MN]
Main value of shear strength for concrete B 35 according to Table 2	τ_{011} = 0.6 [MN/ m^2]
Factor depending on steel grade according to Table 6	α_s = 1.4

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o A_{crit}$$

$$Q_r = 10.49 - 15.323\sigma_o \text{ [MN]}$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{uh}$$

$$\tau_r = \frac{10.49 - 15.323\sigma_o}{8.536 \times 1.7} = 0.723 - 1.056\sigma_o \text{ [MN/ m}^2\text{]}$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sxc} + A_{syc}}{2h}$$

$$\mu_g = \frac{A_{sxc} + A_{syc}}{2 \times 1.7 \times 100} = 0.00294(A_{sxc} + A_{syc}) \text{ [\%]}$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\alpha_s\sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.4\sqrt{0.00294(A_{sxc} + A_{syc})} = 0.0987\sqrt{A_{sxc} + A_{syc}}$$

The allowable concrete punching strength τ_{r1} [MN/ m^2] is given by

$$\tau_{r1} = \kappa_1 \tau_{011}$$

$$\tau_{r1} = 0.0987\sqrt{A_{sxc} + A_{syc}} \times 0.6 = 0.0592\sqrt{A_{sxc} + A_{syc}} \text{ [MN/ m}^2\text{]}$$

Table 41 shows the check of punching shear for the interior column P33 by application of different soil models where for all soil models $\tau_r < \tau_{r1}$. Therefore, the section is safe for punching shear.

Table 41 Check for punching shear by application of different soil models

Soil model		σ_{33} [MN/ m^2]	$A_{sxc} + A_{scx}$ [cm^2/ m]	τ_r [MN/ m^2]	τ_{r1} [MN/ m^2]
Simple assumption model	1	0.221	63.90	0.490	0.473 . τ_r
Winkler's model	3	0.217	73.70	0.494	0.508 > τ_r
Continuum model	6	0.181	98.21	0.532	0.587 > τ_r

7.2.2 Exterior column (column P19)

The critical punching shear section for exterior column is considered at column P19 (Figure 33).

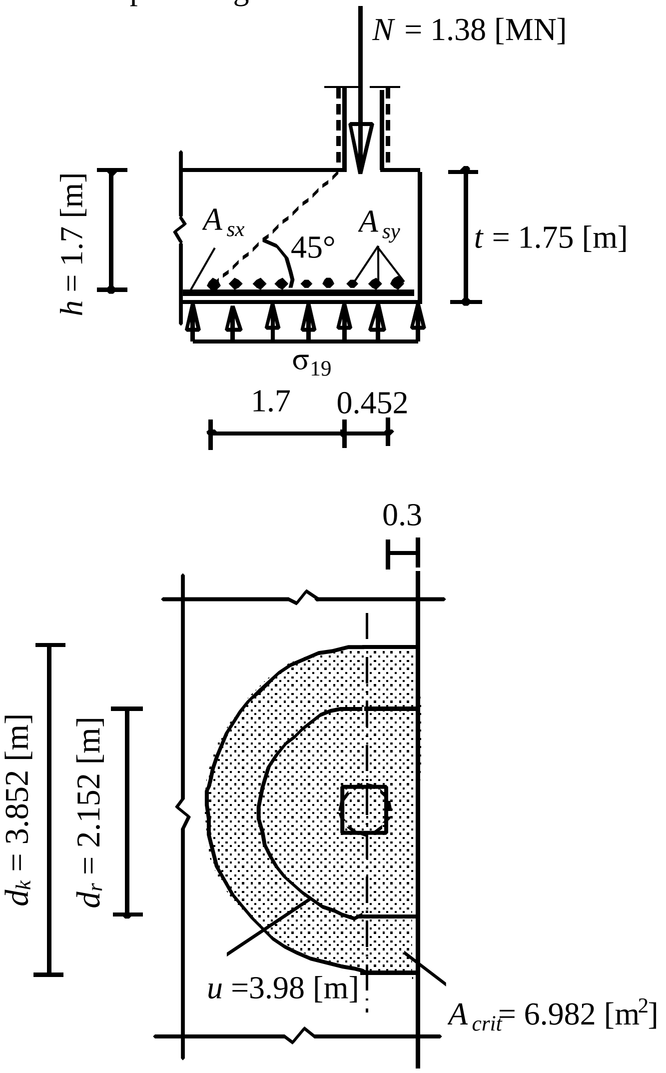


Figure 33 Critical section for punching shear according to DIN 1045

Geometry (Figure 33)

Effective depth of the section	$h = 1.7$ [m]
Column side	$c_x = c_y = 0.4$ [m]
Average diameter of the column	$c = 1.13\ (0.4 \times 0.4)^{1/2} = 0.452$ [m]
Diameter of loaded area	$d_k = 2\ h + c = 2 \times 1.7 + 0.452 = 3.852$ [m]
Diameter of critical punching shear section	$d_r = c + h = 0.452 + 1.7 = 2.152$ [m]
Area of critical punching shear section	$A_{crit} = 0.5\pi d_k^2/\ 4 + 0.3 d_k$
	$A_{crit} = 0.5\pi 3.852^2/\ 4 + 0.3 \times 3.852$
	$= 6.982$ [m^2]
Perimeter of critical punching shear section	$u = 0.5\pi d_r + 2 \times 0.3 = 0.5\pi\ 2.152 + 2 \times 0.3$
	$u = 3.98$ [m]

Loads and stresses

Column load N = 1380 [kN] = 1.38 [MN]
Main value of shear strength for concrete B 35 according to Table 2 τ_{011} = 0.6 [MN/ m^2]
Factor depending on steel grade according to Table 6 α_s = 1.4

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o A_{crit}$$

$$Q_r = 1.38 - 6.982\sigma_o \text{ [MN]}$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{uh}$$

$$\tau_r = \frac{1.38 - 6.982\sigma_o}{3.98 \times 1.7} = 0.204 - 1.032\sigma_o \text{ [MN/ m}^2\text{]}$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sxc} + A_{syc}}{2h}$$

$$\mu_g = \frac{2minA_s}{2 \times 1.7 \times 100} = \frac{2 \times 29.5}{2 \times 1.7 \times 100} = 0.174[\%]$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\alpha_s\sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.4\sqrt{0.174} = 0.759$$

The allowable concrete punching strength τ_{r1} [MN/ m^2] is given by

$$\tau_{r1} = \kappa_1 \tau_{011}$$

$$\tau_{r1} = 0.759 \times 0.6 = 0.456 \text{[MN/ m}^2\text{]}$$

Table 42 shows the check of punching shear for the exterior column P19 by application of different soil models where for all soil models $\tau_r < \tau_{r1}$. Therefore, the section is safe for punching shear.

Table 42 Check for punching shear by application of different soil models

Soil model		σ_{19} [MN/ m^2]	A_s [cm^2/ m]	τ_r [MN/ m^2]	τ_{r1} [MN/ m^2]
Simple assumption model	1	0.145	29.5	0.054	0.456 > τ_r
Winkler's model	3	0.164	29.5	0.035	0.456 > τ_r
Continuum model	6	0.159	29.5	0.040	0.456 > τ_r

8 Comparison between the design according to DIN 1045 and EC 2

Table 43 to Table 46 show the comparison between the design of a raft according to DIN 1045 and EC 2 by application of different soil models. The comparison is considered only for required reinforcement due to flexure moment at the critical sections *x-x* and *y-y*.

From the comparison the following can be concluded:

- If the raft has the same thickness and is designed according to EC 2 and DIN 1045, the reinforcement obtained from EC 2 will be less than that obtained from DIN 1045 by 9 [%]

- For Continuum model, the contact pressure values at the edges of the raft are higher than those at the middle. Consequently, the positive moment under the column P33 for Continuum model is higher than that for both Simple assumption and *Winkler's* models, while the negative moment in the filed is less than that of the other models. This relation is valid also for reinforcement

- The contact pressure for Simple assumption and *Winkler's* models are quite similar, particularly if the soil is uniform. Therefore, the results of both models are nearly the same

- If the reinforcement under the column decreases, the reinforcement in the field will increase and vice verse. This notice yields for a constant amount of reinforcement in the section. However, the difference in reinforcement calculated by the three models is about 40 [%]. The design of the raft by all methods is considered acceptable in this example

Table 43 Comparison between the design according to DIN 1045 and EC 2 for required bottom reinforcement A_{sxc} under the column at section *x-x*

Soil model		A_{sxc} [cm^2/ m] according to		Difference ΔA_{sxc} [%]
		DIN 1045	EC 2	
Simple assumption model	1	30.35	27.89	8.82
Winkler's model	3	38.62	35.50	8.79
Continuum model	6	49.41	45.40	8.83

Table 44 Comparison between the design according to DIN 1045 and EC 2 for required top reinforcement A_{sxf} in the field at section *x-x*

Soil model		A_{sxf} [cm^2/ m] according to		Difference ΔA_{sxf} [%]
		DIN 1045	EC 2	
Simple assumption model	1	46.37	42.62	8.80
Winkler's model	3	36.47	33.52	8.80
Continuum model	6	27.09	24.89	8.84

Table 45 Comparison between the design according to DIN 1045 and EC 2 for required bottom reinforcement A_{syc} under the column at section *y-y*

Soil model		A_{syc} [cm^2/ m] according to		Difference ΔA_{syc} [%]
		DIN 1045	EC 2	
Simple assumption model	1	29.92	27.49	8.84
Winkler's model	3	32.13	29.53	8.81
Continuum model	6	39.47	36.27	8.82

Table 46 Comparison between the design according to DIN 1045 and EC 2 for required top reinforcement A_{syf} in the field at section *y-y*

Soil model		A_{syf} [cm^2/ m] according to		Difference ΔA_{syf} [%]
		DIN 1045	EC 2	
Simple assumption model	1	23.79	21.86	8.83
Winkler's model	3	21.43	19.69	8.84
Continuum model	6	14.43	13.25	8.91

9 References

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