Example 4: Design of a circular raft for a cylindrical core

1 Description of the problem

Ring or circular rafts can be used for cylindrical structures such as chimneys, silos, storage tanks, TV-towers and other structures. In this case, ring or circular raft is the best suitable foundation to the natural geometry of such structures. The design of circular rafts is quite similar to that of other rafts.

As a design example for circular rafts, consider the cylindrical core wall shown in Figure 34 as a part of five-story office building. The diameter of the core wall is 8.0 [m], while the width of the wall is B = 0.3 [m]. The core lies in the center of the building and it does not subject to any significant lateral applied loading. Therefore, the core wall carries only a vertical load of p = 300 [kN/ m]. The base of the cylindrical core wall is chosen to be a circular raft of 10.0 [m] diameter with 1.0 [m] ring cantilever. A thin plain concrete of thickness 0.15 [m] is chosen under the raft and is unconsidered in any calculations.

Two analyses concerning the effect of wall rigidity on the raft are carried out in the actual design, both by using the Continuum model (method 6) to represent the subsoil. The two cases of analyses are considered as follows:

Case 1: The presence of the core wall is ignored

Case 2: A height of only one storey is taken into account, where the perimeter wall is modeled by beams having the flexural properties of B = 0.3 [m] width and H = 3.0 [m] height. The choice of this reduced wall height is because the wall above the first floor has many openings

Figure 34 shows plan of the raft, wall load, dimensions and mesh with section through the raft and subsoil. The following text gives a description of the design properties and parameters.

2 Properties of the raft material

Young's modulus of concrete	E_b	$= 3.2 \times 10^7$	[kN/ m^2]
Poisson's ratioof concrete	ν_b	= 0.20	[-]
Shear modulus of concrete	$G_b = 0.5\, E_b\,(1+\nu_b)$	$= 1.3 \times 10^7$	[kN/ m^2]
Unit weight of concrete	γ_b	= 25	[kN/ m^3]

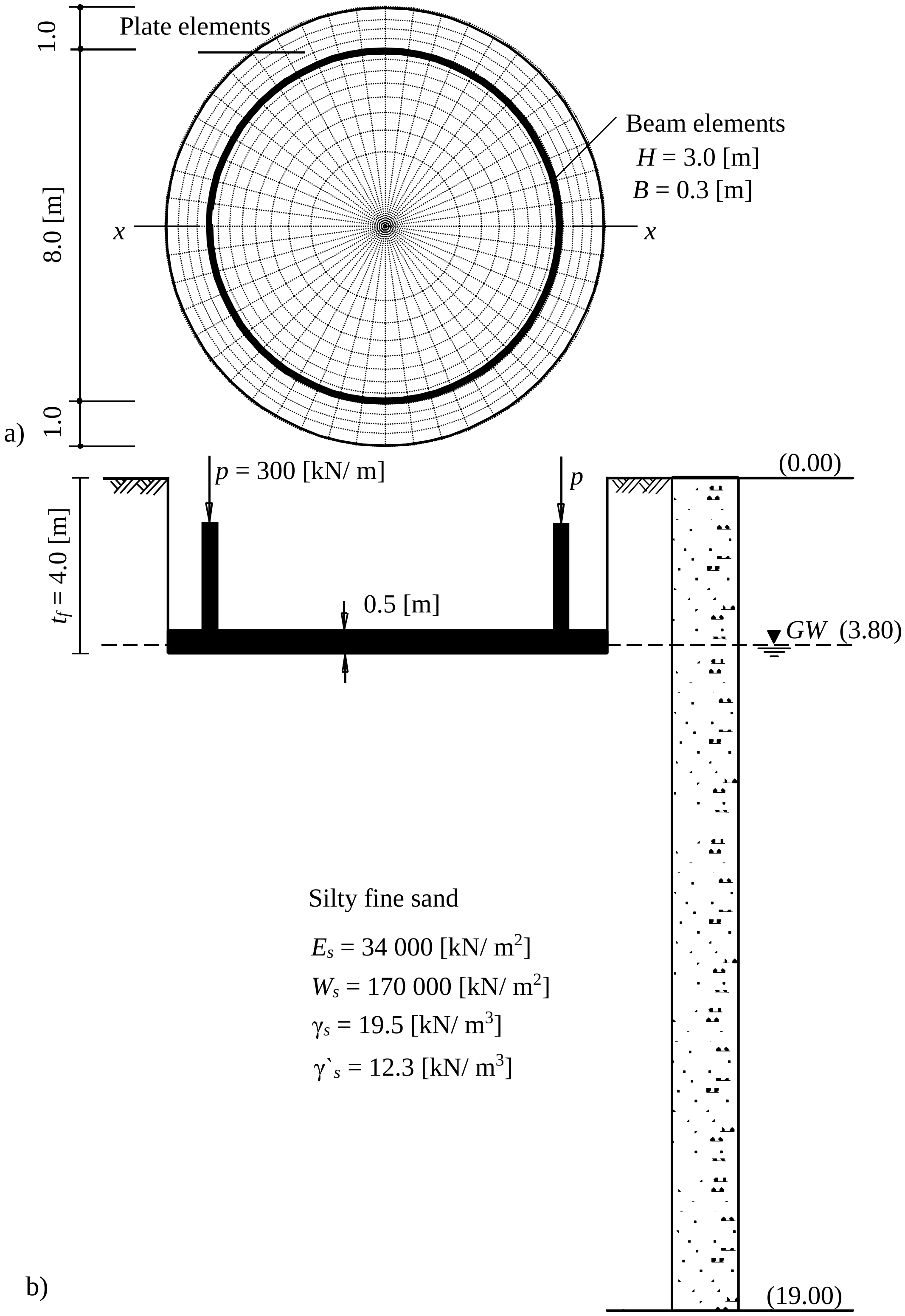

Figure 34 a) Plan of the raft with wall load, dimensions and mesh
b) Section through the raft and subsoil

3 Properties of the raft section

The raft section has the following parameters:

Width of the section to be designed	$b = 1.0$	[m]
Section thickness	$t = 0.50$	[m]
Concrete cover + 1/2 bar diameter	$c = 5$	[cm]
Effective depth of the section	$d = t - c = 0.45$	[m]
Steel bar diameter	$\Phi = 14$	[mm]

Minimum area of steel per meter, $A_s\ min$ is

$A_s\ min = 0.15\ [\%] \times \text{concrete section} = 0.0015 \times 50 \times 100 = 7.5\ [\text{cm}^2/\ \text{m}]$

take min $A_s\ min = 6\ \Phi\ 25 = 29.5\ [\text{cm}^2/\ \text{m}]$

4 Soil properties

The core rests on a soil layer of 15.0 [m] of silty fine sand, overlying a rigid base of sandstone as shown in Figure 34. The effect of uplift pressure, reloading of the soil and limit depth of the soil layer are taken into account. The soil layer has the following parameters:

Poisson's ratio	$\nu_s = 0.30$	[-]
Level of foundation depth under the ground surface	$d_f = 4.0$	[m]
Modulus of compressibility for loading	$E_s = 34\ 000$	[kN/ m^2]
Modulus of compressibility for reloading	$W_s = 170\ 000$	[kN/ m^2]
Unit weight above the ground water	$\gamma_s = 19.5$	[kN/ m^3]
Unit weight under the ground water	$\gamma'_s = 12.3$	[kN/ m^3]
Level of water table underground surface	$GW = 3.8$	[m]

5 Analysis of the raft

The raft is subdivided into 576 quadrature and triangular elements. Then, the analysis of the raft is carried out two times for two different structural systems. In the first analysis, the rigidity of the core wall is ignored and only the self-rigidity of 0.5 [m] raft thickness is considered. In the other analysis, the rigidity of the core wall is considered through inserting additional beam elements along the location of the wall on the FE-mesh. The properties of the beam elements (width $B = 0.3$ [m], height $H = 3.0$ [m]) are:

Moment of Inertia $I = BH^3/\ 12 = 0.675\ [\text{m}^4]$

Torsional Inertia $$J = HB^3\left(\frac{1}{3} - 0.21\frac{B}{H}\right)\left(1 - \frac{B^4}{12H^4}\right) = 0.0253[\text{m}^4]$$

To make better representation for the line loads on the raft, the loads from the wall are modeled as uniform loads acting on the beam elements. In case of the structural system without effect of the wall, beam elements may be remaining in the system while the rigidity of the wall is eliminated by defining all property values of the beam elements by zero except the loads. The system of linear equations for the Continuum model is solved by iteration (method 6). The system of linear equations for the Continuum model is solved by iteration (method 6). The maximum difference between the soil settlement *s* [cm] and the raft deflection *w* [cm] is considered as an accuracy number. In this example, the accuracy is chosen $\varepsilon = 0.0002$ [cm]. Another element mesh type can be used for the raft, where in this case the raft is subdivided into 404 rectangular elements as shown in Figure 35. Both the two finite element meshes of Figure 34 and Figure 35, give nearly the same results. The presented results here are for the second mesh of only rectangular elements, which are calculated by an earlier version of ELPLA. The data folder of this example contains files of the two finite element meshes.

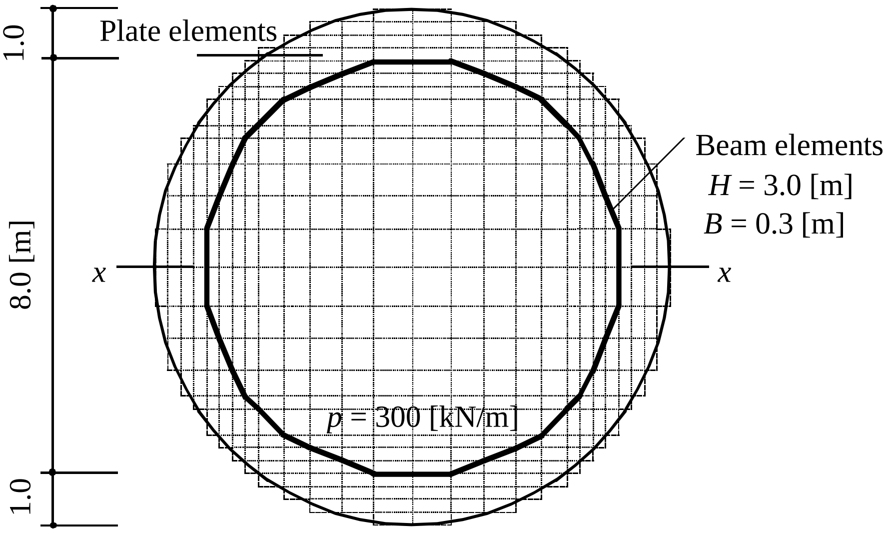

Figure 35 Rectangular finite element mesh

Determination of the limit depth t_s

The level of the soil under the raft in which no settlement occurs or the expected settlement will be very small where can be ignored is determined first as a limit depth of the soil. The limit depth in this example is chosen to be the level of which the stress due to the raft σ_E reaches the ratio $\xi = 0.2$ of the initial vertical stress σ_V. The stress in the soil σ_E is determined at the characteristic point *c* of the circular foundation. This stress σ_E is due to the average stress from the raft at the surface $\sigma_O = 108$ [kN/ m^2]. At the characteristic point, from the definition of *Graßhoff* (1955), the settlement if the raft is full rigid will be identical with that if the raft is full flexible. The characteristic point *c* lies at a distance 0.845 *r* from the center of the raft as shown in Figure 36. The results of the limit depth calculation are plotted in a diagram as shown in Figure 36. The limit depth is found to be $t_s = 11.23$ [m] under the ground surface.

6 Evaluation and conclusions

To evaluate the analysis results, the results of both analyses are compared together. The following conclusions are drawn:

Settlements

Figure 37 and Figure 38 show the extreme values of settlements in *x*-direction under the raft, while Figure 39 shows the settlements at section *x-x* under the middle of the raft for both cases of analyses. From the figures, it can be concluded the following:

- The maximum differential settlement across the raft without the effect of the wall (Δs = 0.2 [cm]) is double that with effect of the wall (Δs = 0.1 [cm])

- The maximum settlements, if the presence of the wall is considered, decrease from 0.49 [cm] to 0.45 [cm] by 9 [%], while the minimum settlements, if the presence of the wall is considered, increase from 0.29 [cm] to 0.35 [cm] by 21 [%]

- The presence of the wall improves the deformation shape where the settlements at the raft edges will decrease, while those at the center will increase

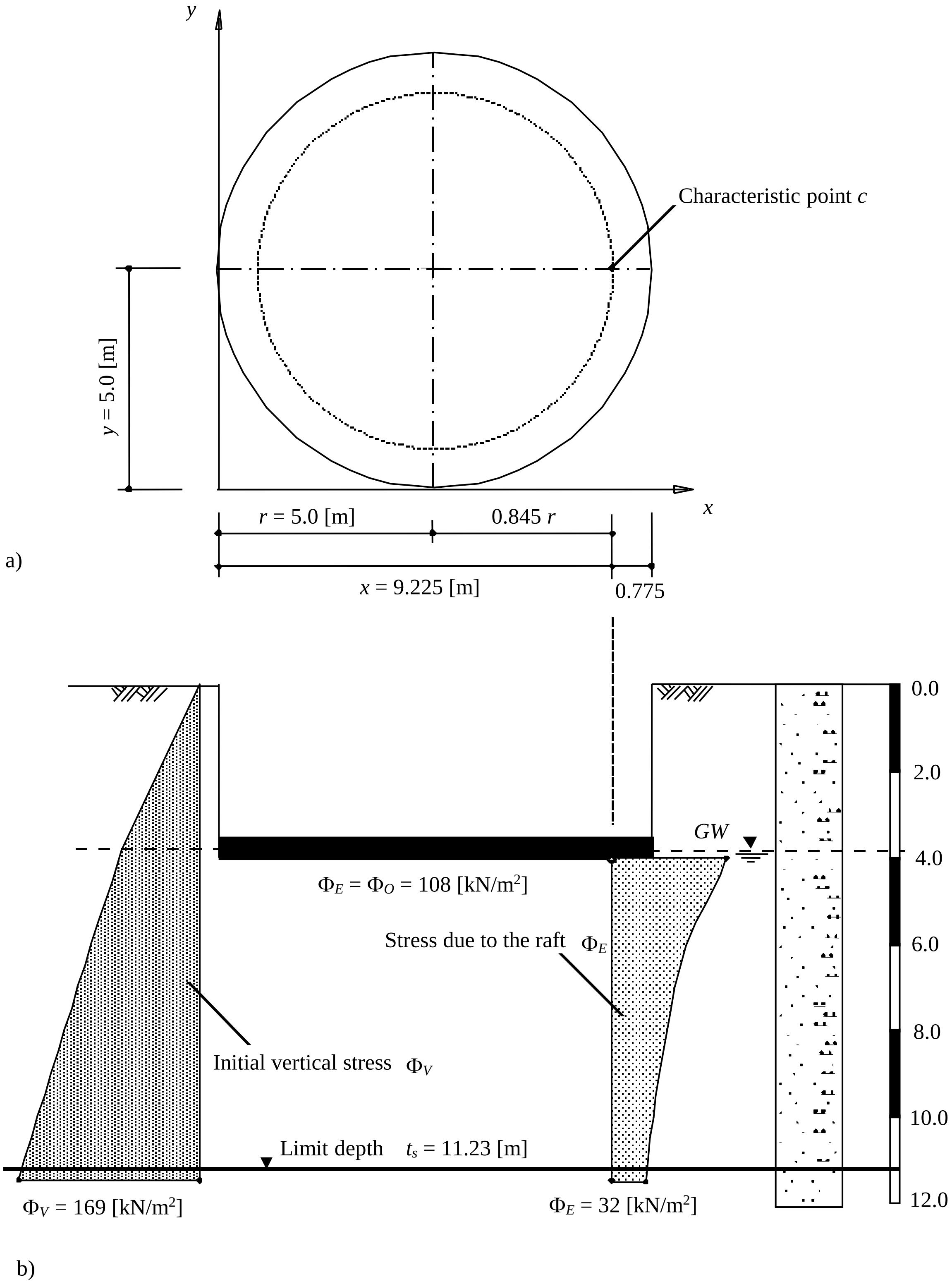

Figure 36 a) Position of characteristic point *c*
b) Limit depth t_s of the soil under the raft

Contact pressures

Figure 40 shows the contact pressures *q* at section *x*-*x* for both analyses without and with effect of the wall.

- The difference in contact pressures for both analyses is not great along the raft, only a slight difference is found at the center and the edge of the raft
- If the entire distribution of contact pressure is taken to be uniform (108 [kN/ m^2]), in the manner frequently assumed in traditional foundation design, the negative moments will be much higher, while the positive moment will be lower (not shown)

Moments

As the circular raft is a special case of rafts, radial moments m_r are equal to both principal moments h_{m1} and moments m_x in *x*-direction at the section pass through the center of the raft. In addition, tangential moments m_t are equal to both principal moments h_{m2} and moments m_y in *y*-direction at the section pass through the center of the raft. Figure 41 to Figure 44 show the contour lines of radial and tangential moments, while Figure 45 and Figure 46 show the vectors of principal moments $h_{m1,\ 2}$ of the raft for both analyses. Figure 47 shows the radial and tangential moments in one figure at section *x*-*x*. These results show that:

- The absolute values of negative radial and tangential moments m_r and m_t at the center of the raft in the analysis with effect of the wall ($m_r = m_t = -95$ [kN.m/ m]) are lower than that in the analysis without effect of the wall ($m_r = m_t = -124$ [kN.m/ m]) by 31 [%]. Therefore, the positive moments m_r and m_t under the wall increase due to taking of wall effect in the analysis
- The positive radial moments m_r under the wall increase from 98 [kN.m/ m] to 130 [kN.m/ m] due to taking of wall effect in the analysis by 25 [%]
- Positive tangential moments will occur only, if the analysis considers effect of the wall

Table 47 shows a comparison of the results at the critical sections for the raft without and with effect of the wall, which recommends the above conclusions.

Table 47 Settlements, contact pressures, radial and tangential moments at critical sections of the raft for both analyses without and with the effect of wall

Results	Position	Presence of the wall		Difference Δ [%]
		is ignored	is considered	
Settlements s [cm]	Edge	0.49	0.45	9
	Center	0.29	0.35	17
	Under the wall	0.46	0.45	2
Contact pressures q [kN/ m^2]	Edge	340	299	14
	Center	71	78	9
	Under the wall	102	102	0
Radial moments m_r [kN.m/ m]	Edge	0.0	3	100
	Center	-125	-95	31
	Under the wall	98	130	25
Tangential moments m_t [kN.m/ m]	Edge	-17	0.0	-
	Center	-125	-96	30
	Under the wall	-15	22	168

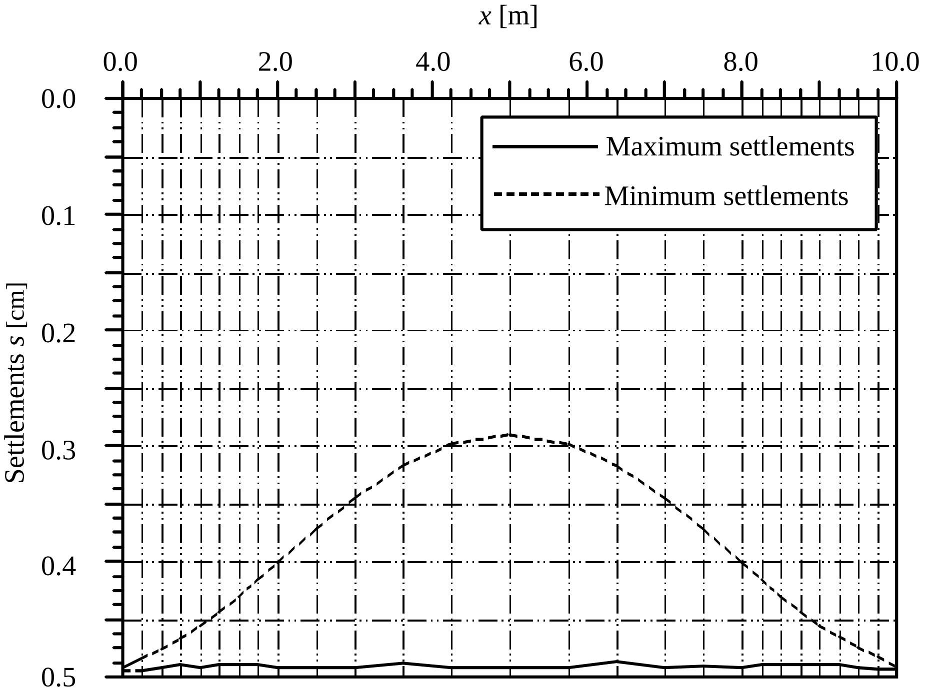

Figure 37 Extreme values of settlements *s* [cm] in *x*-direction under the raft without effect of the wall

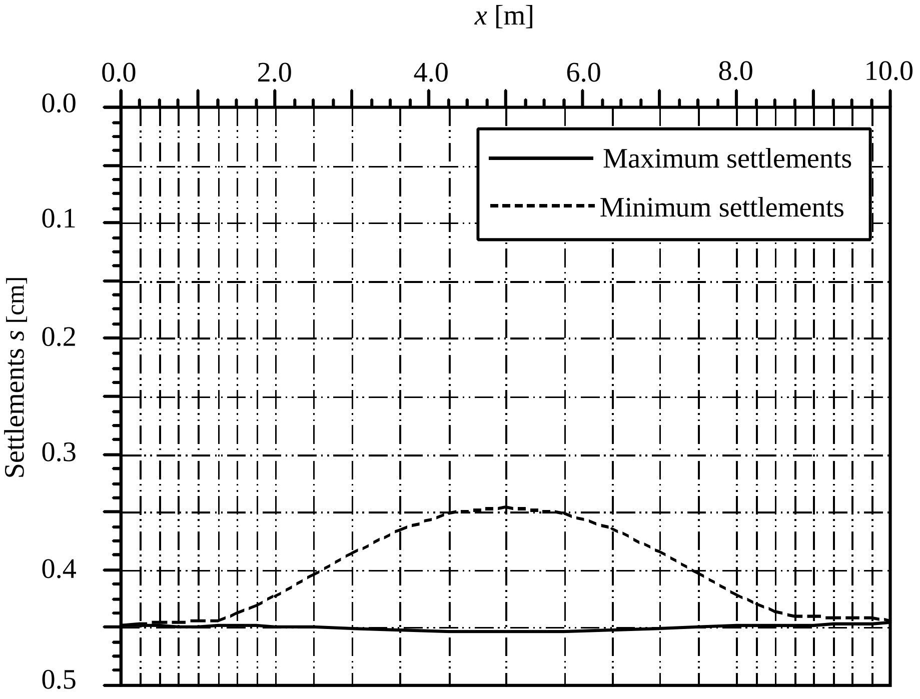

Figure 38 Extreme values of settlements *s* [cm] in *x*-direction under the raft with effect of the wall

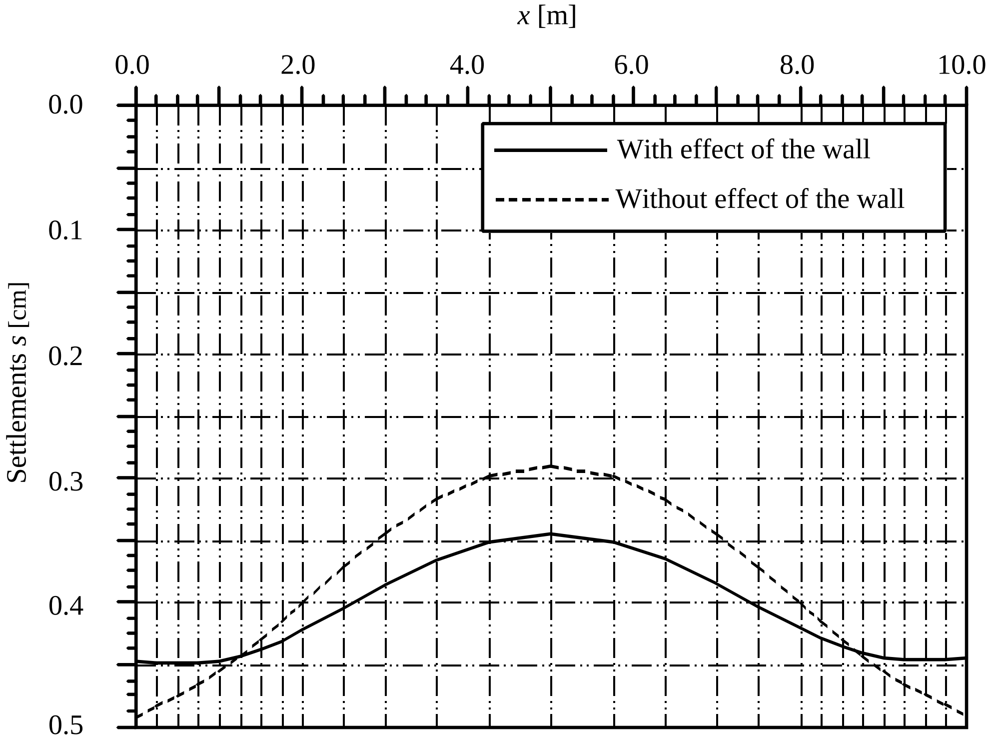

Figure 39 Settlements s [cm] at section x-x

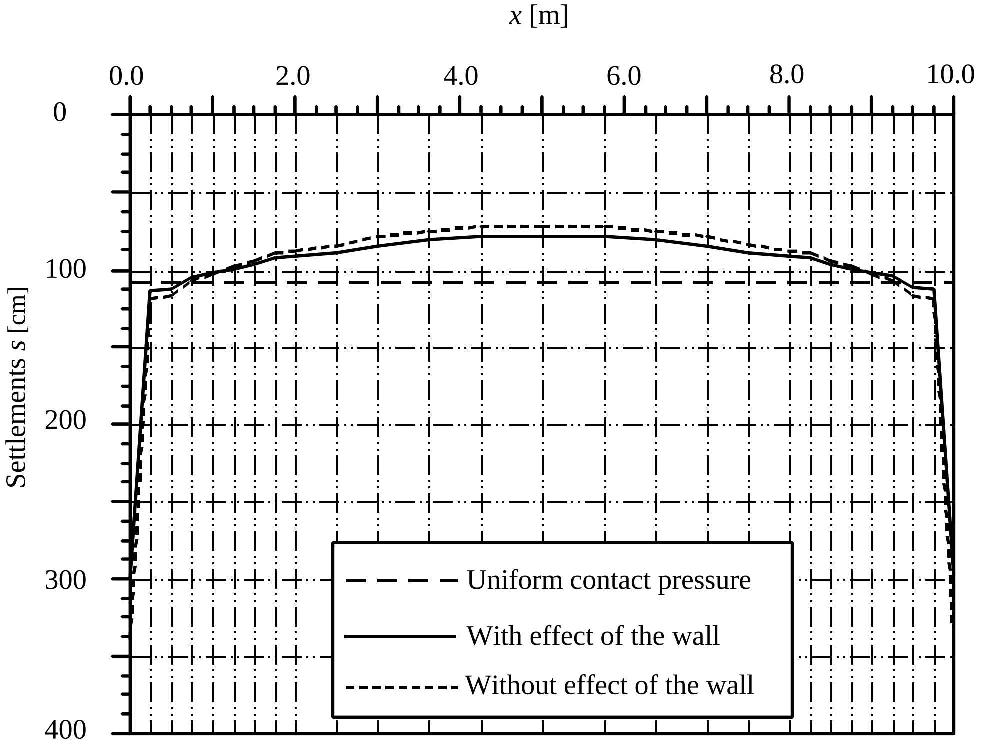

Figure 40 Contact pressures q [kN/ m^2] at section x-x

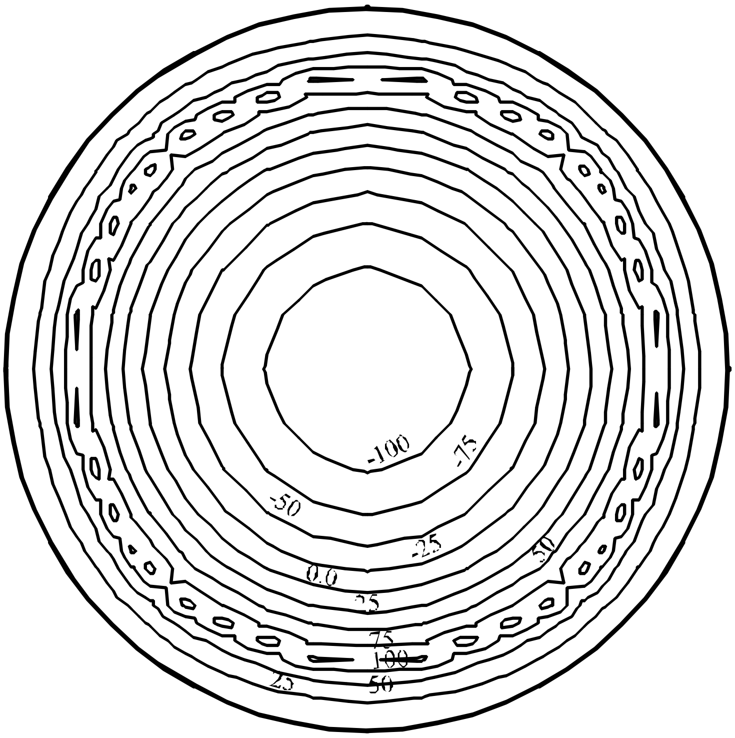

Figure 41 Contour lines of radial moments $m_r = h_{m1}$ [kN.m/ m] of the raft without effect of the wall

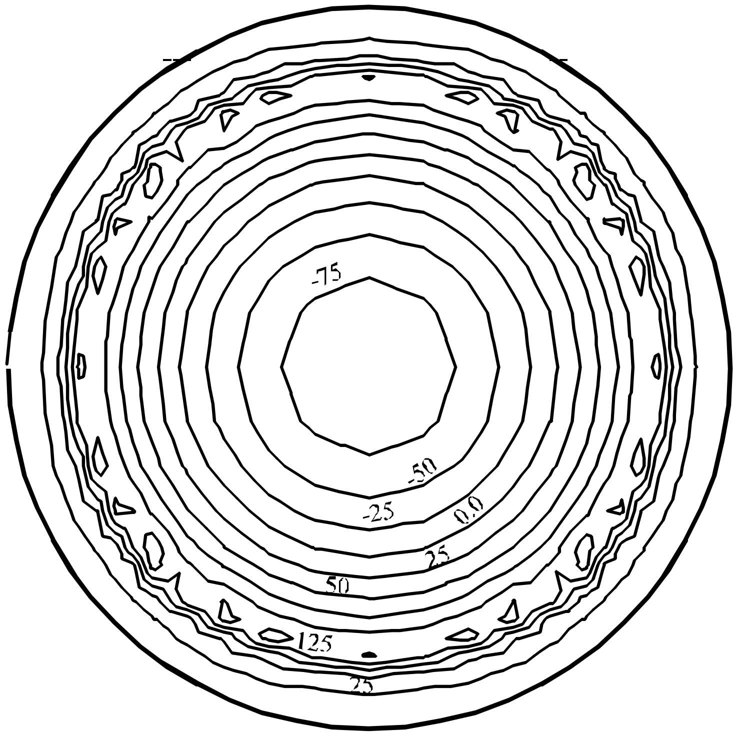

Figure 42 Contour lines of radial moments $m_r = h_{m1}$ [kN.m/ m] of the raft with effect of the wall

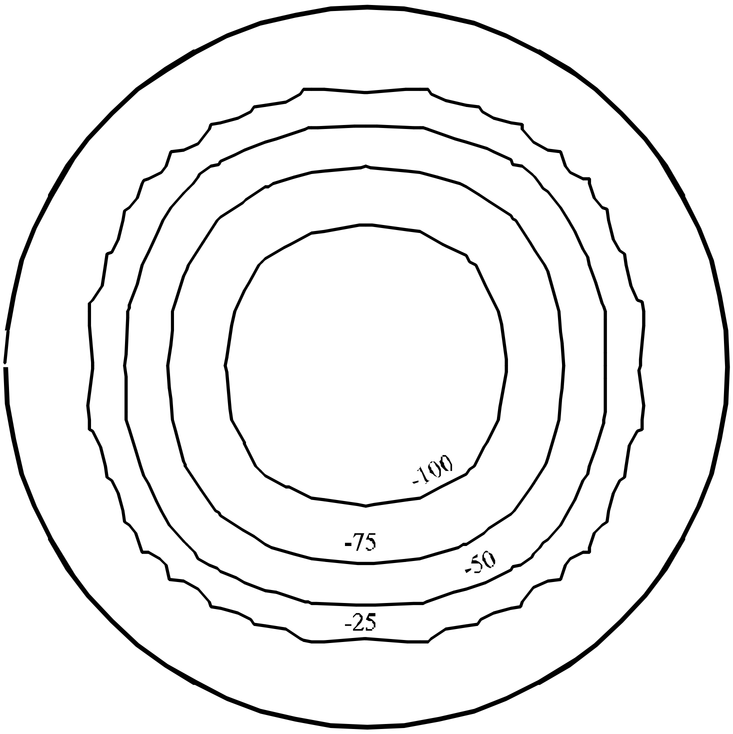

Figure 43 Contour lines of tangential moments $m_t = h_{m2}$ [kN.m/ m] of the raft without effect of the wall

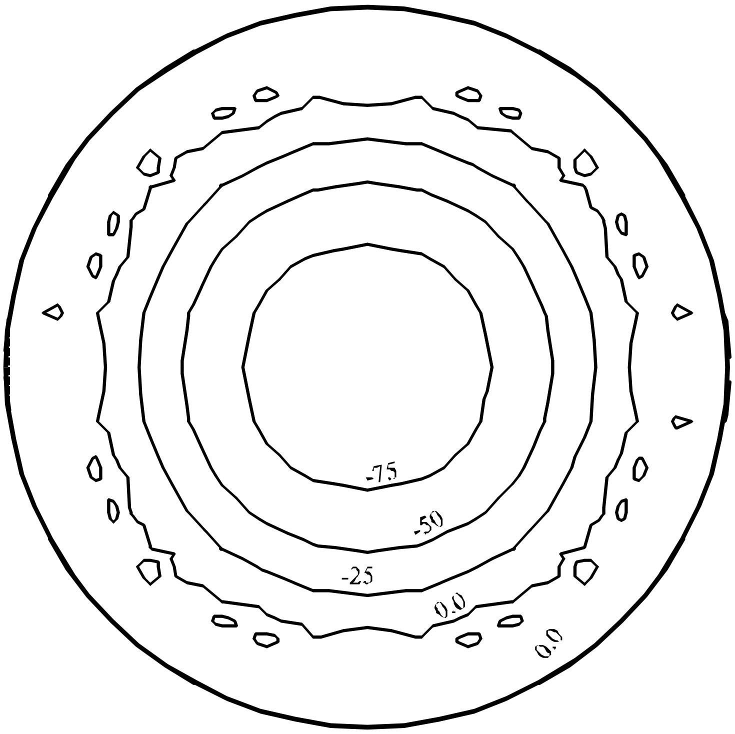

Figure 44 Contour lines of tangential moments $m_t = h_{m2}$ [kN.m/ m] of the raft with effect of the wall

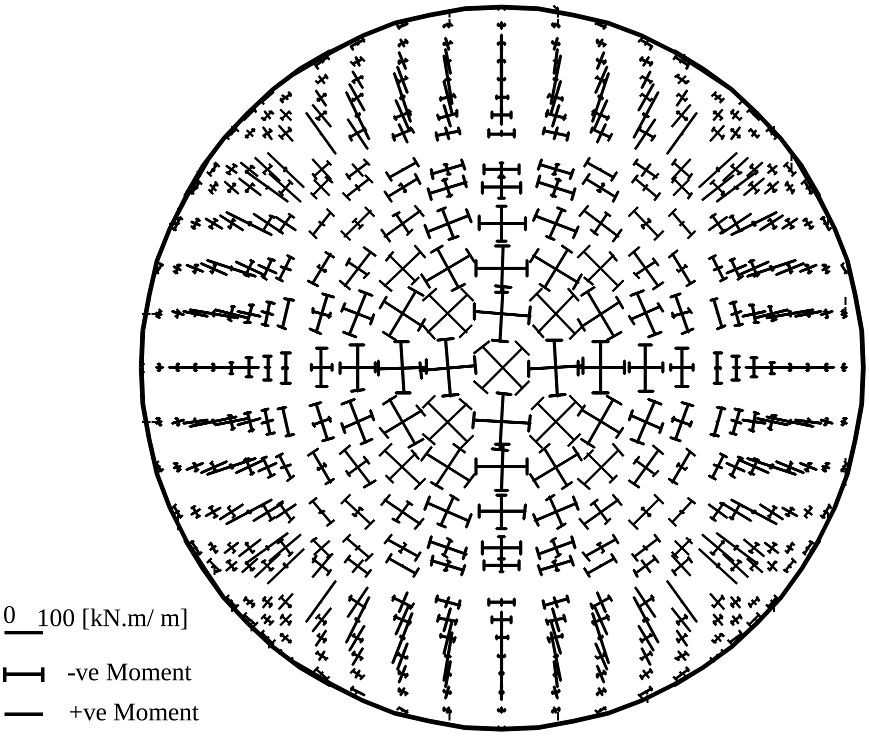

Figure 45 Vectors of principal moments h_{m1} and h_{m2} [kN.m/ m] of the raft without effect of the wall

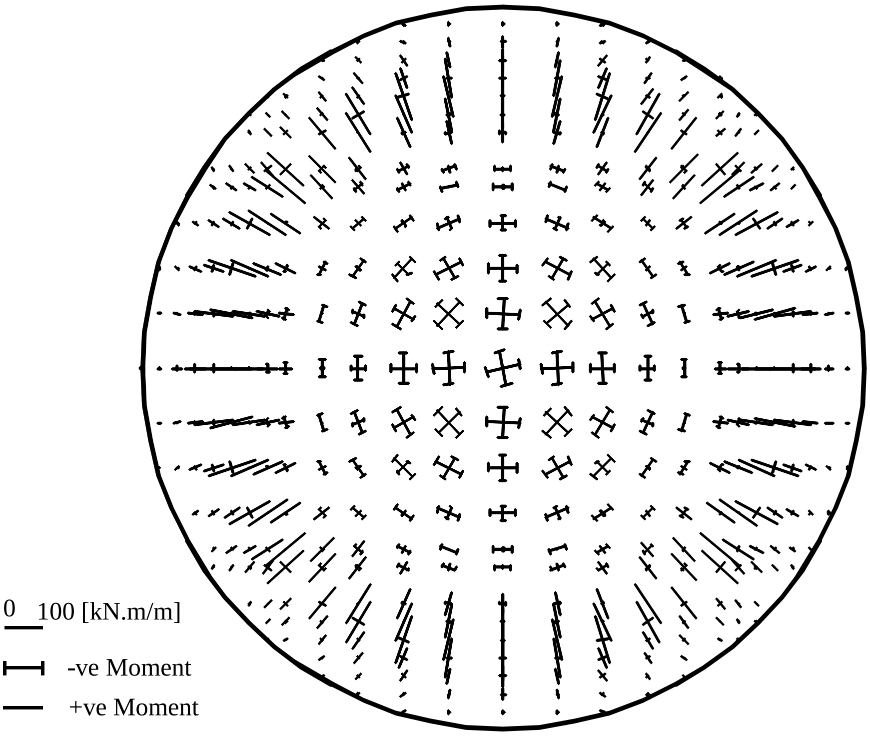

Figure 46 Vectors of principal moments h_{m1} and h_{m2} [kN.m/ m] of the raft with effect of the wall

7 Design of the raft for flexure moment according to EC 2

Material

Concrete grade	C 30/37
Steel grade	BSt 500
Characteristic compressive cylinder strength of concrete	$f_{ck} = 30$ [MN/ m^2]
Characteristic tensile yield strength of reinforcement	$f_{yk} = f_y = 500$ [MN/ m^2]
Partial safety factor for concrete strength	$\gamma_c = 1.5$
Design concrete compressive strength	$f_{cd} = f_{ck}/ \gamma_c = 30/ 1.5 = 20$ [MN/ m^2]
Partial safety factor for steel strength	$\gamma_s = 1.15$
Design tensile yield strength of reinforcing steel	$f_{yd} = f_{yk}/ \gamma_s = 500/ 1.15 = 435$ [MN/ m^2]

Factored moment

Total load factor for both dead and live loads	$\gamma = 1.395$
Factored column moment	$M_{sd} = \gamma m_r$
Factored field moment	$M_{sd} = \gamma m_t$

Geometry

Effective depth of the section	$d = 0.45$ [m]
Width of the section to be designed	$b = 1.0$ [m]

Determination of tension reinforcement

The design of sections is carried out for EC 2 in table forms. Table 48 to Table 50 and Figure 47 show the design of critical sections.

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 0.45^2 (0.85 \times 20)} = 0.2905 M_{sd}$$

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.2905 M_{sd}} = 1 - \sqrt{1 - 0.581 M_{sd}}$$

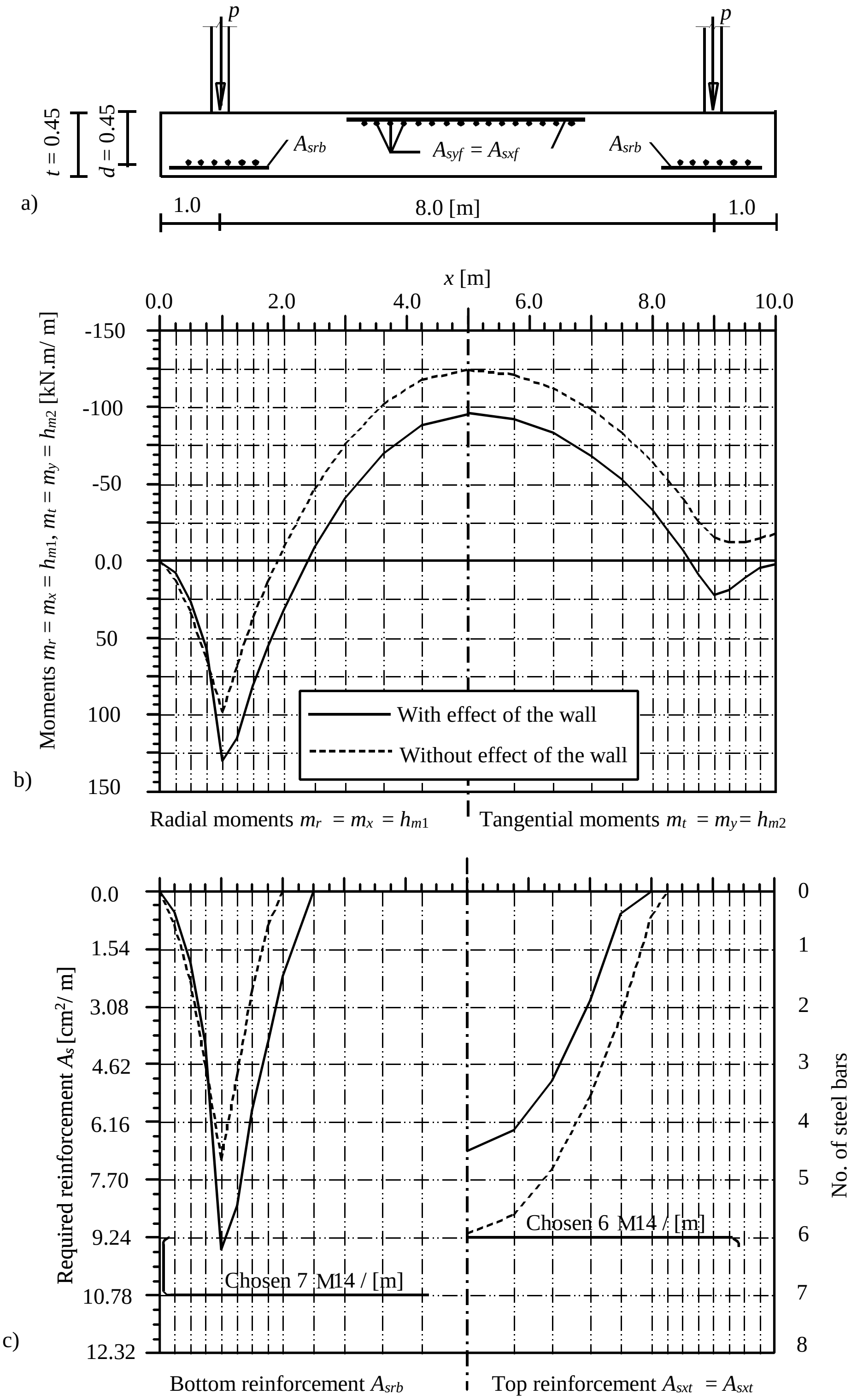

Figure 47 a) Section *x*-*x* through the raft
b) Moments $m_r = m_x = h_{m1}$, $m_t = m_y = h_{m2}$ [kN.m/ m] at section *x*-*x*
c) Main reinforcement A_s at critical sections

The required area of steel reinforcement per meter A_s is

$$A_s = \omega\left(\frac{(0.85 f_{cd})bd}{f_{yd}}\right)$$

$$A_s = \omega\left(\frac{(0.85\times 20)\times 1.0\times 0.45}{435}\right) = 0.017586\omega\,[\mathrm{m}^2/\,\mathrm{m}]$$

$$A_s = 175.86\omega\ [\mathrm{cm}^2/\,\mathrm{m}]$$

Table 48 Required bottom reinforcement in radial direction A_{srb} for the raft without and with effect of the wall

Structural system	M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{srb} [cm^2/ m]
Raft without effect of the wall	0.137	0.0397	0.0405	7.13
Raft with effect of the wall	0.181	0.0527	0.0542	9.52

Table 49 Required bottom reinforcement in tangential direction A_{stb} for the raft without and with effect of the wall

Structural system	M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{stb} [cm^2/ m]
Raft without effect of the wall	-	-	-	-
Raft with effect of the wall	0.0307	0.009	0.009	1.58

Table 50 Required top reinforcement in the field A_{sxf} = A_{syf} for the raft without and with effect of the wall (both *x*- and *y*-directions)

Structural system	M_{sd} [MN.m/ m]	μ_{sd}	ω	A_{stb} [cm^2/ m]
Raft without effect of the wall	0.174	0.0507	0.0521	9.15
Raft with effect of the wall	0.133	0.0385	0.0393	6.91

Chosen reinforcement

Table 51 shows the chosen reinforcement for the raft. The bottom reinforcement is chosen to be in radial and tangential directions while the top reinforcement is chosen to be in *x*- and *y*-directions. The design considers the maximum reinforcement obtained from both the analyses of the two structural systems. The chosen diameter of steel bars is Φ = 14 [mm].

Table 51 Chosen reinforcement

Bottom reinforcement		Top reinforcement in *x*- and *y*-directions $A_{sxt} = A_{syt}$
Radial direction A_{srb}	Tangential direction A_{stb}	
7 Φ 14 = 10.78 [cm^2/ m]	$min\ A_s$ = 7.7 [cm^2/ m]	6 Φ 14 = 9.24 [cm^2/ m]

According to the design of the raft for two structural systems, the raft is reinforced by a square mesh 6 Φ 14 [mm/ m] in the upper surface, while the lower surface is reinforced by 7 Φ 14 [mm/ m] in radial direction and 5 Φ 14 [mm/ m] in tangential direction. In addition, an upper radial and tangential reinforcement 5 Φ 14 [mm/ m] is used at the cantilever ring. A small square mesh 5 Φ 14 [mm/ m], each side is 1.0 [m], is used at the center of the raft to connect the bottom radial reinforcement. The details of reinforcement of the raft are shown in Figure 48.

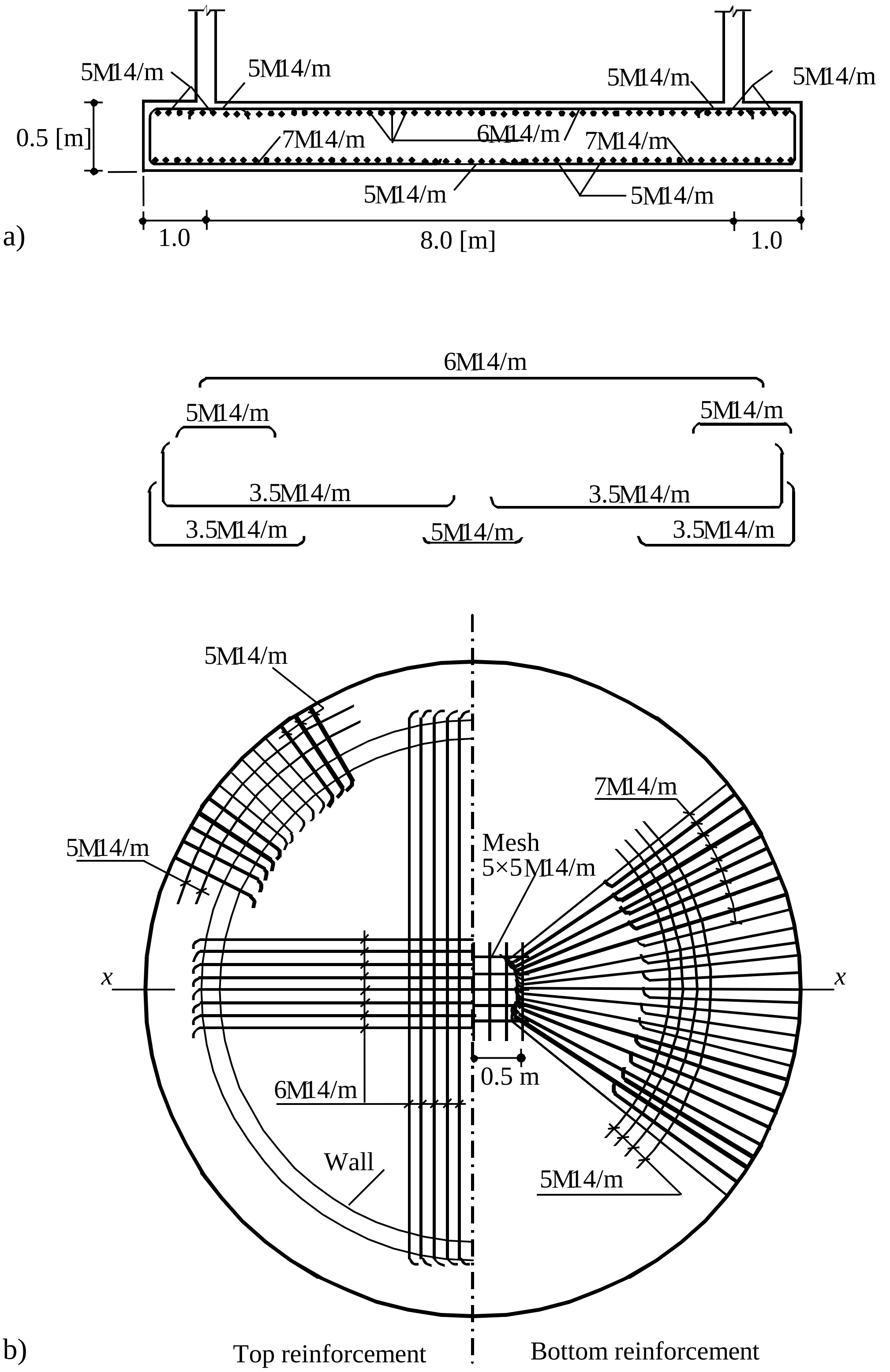

Figure 48 a) Section *x*-*x* through the raft with reinforcement
b) Reinforcement of the raft in plan