## Example 5: Comparison between flat and ribbed rafts

### 1 Description of the problem

A ribbed raft may be used where the distance between columns is so great that a flat raft requires excessive depth, with resulting high bending moments. Consequently, the volume of concrete is reduced. A ribbed raft consists of a stiffened slab by girders in *x*- and *y*-directions. The girders on the raft may be either down or up the slab. Ribbed rafts can be used for many structures when a flat level for the first floor is not required. Such structures are silos, elevated tanks and various other possible structures. Although this type of foundation has many disadvantages if used in normally buildings, still is used by many designers. Such disadvantages are: the raft needs deep foundation level under the ground surface, fill material on the raft to make a flat level. In addition, a slab on the fill material is required to be constructed for the first floor. The use of the ribbed raft relates to its simplicity in analysis by traditional manners or hand calculations, particularly, if the columns are arranged in lines. The ribbed raft generally leads to less concrete quantity than the flat raft, especially if the columns have heavy loads and large spans.

In this example two types of rafts, flat and ribbed rafts, are considered as shown in Figure 49 The length of each raft is $L$ = 14.3 [m] while the width is $B$ = 28.3 [m]. Each raft carries 15 column loads and a brick wall load of $p$ = 30 [kN/ m] at its edges. Width of ribs is chosen to be $b_w$ = 0.30 [m] equal to the minimum side of columns, while the height of ribs including the slab thickness is chosen to be $h_w + h_f$ = 1.0 [m]. Column dimensions, reinforcement and loads are shown in Table 52. A thin plain concrete of thickness 0.20 [m] is chosen under the raft and is not considered in any calculation.

Table 52 Column models, loads, dimensions and reinforcement

| Column | Load [kN] | Dimensions [m × m] | Reinforcement |
|---|---|---|---|
| Model C1 | 781 | 0.30 × 0.30 | 6 Φ 16 |
| Model C2 | 1562 | 0.30 × 0.70 | 4 Φ 16 + 4 Φ 19 |
| Model C3 | 3124 | 0.30 × 1.40 | 6 Φ 22 + 6 Φ 19 |

Two analyses are carried out to compare between the two structural systems of rafts. In the analyses, the Continuum model is used to represent the subsoil. The two cases of analyses are considered as follows:

- Flat raft for optimal raft thickness
- Ribbed raft for optimal slab thickness

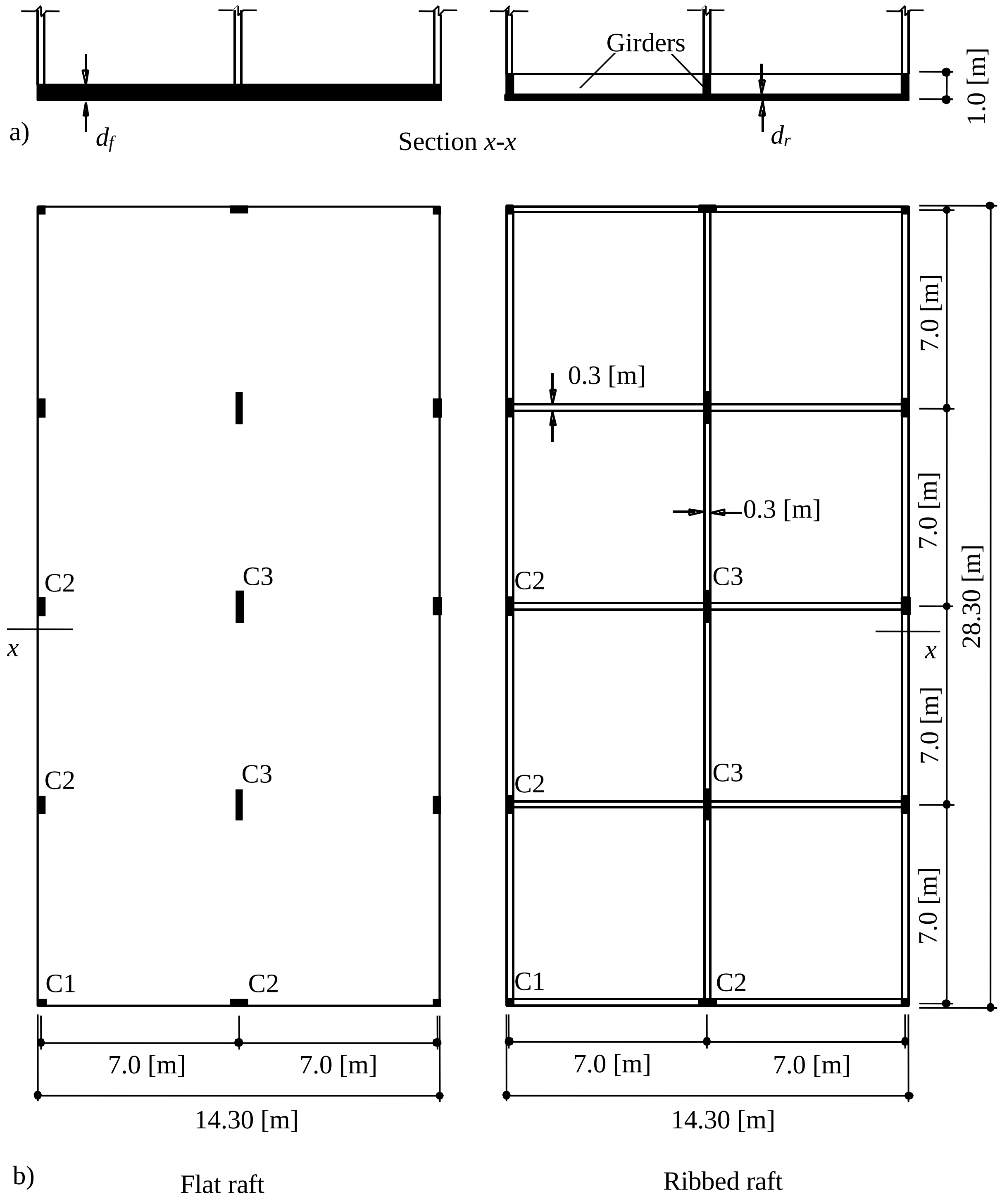


Figure 49 a) Plan of rafts and dimensions
b) Section through the rafts

## 2 Properties of the raft material

The material of rafts is reinforcement concrete that has the following parameters:

| | | |
|---|---|---|
| *Young's* modulus of concrete | $E_b$ | $= 3.2 \times 10^7$ [kN/ m$^2$] |
| *Poisson's* ratioof concrete | $\nu_b$ | $= 0.20$ [-] |
| Shear modulus of concrete | $G_b = 0.5\, E_b\, (1 + \nu_b)$ | $= 1.3 \times 10^7$ [kN/ m$^2$] |
| Unit weight of concrete | $\gamma_b$ | $= 25$ [kN/ m$^3$] |

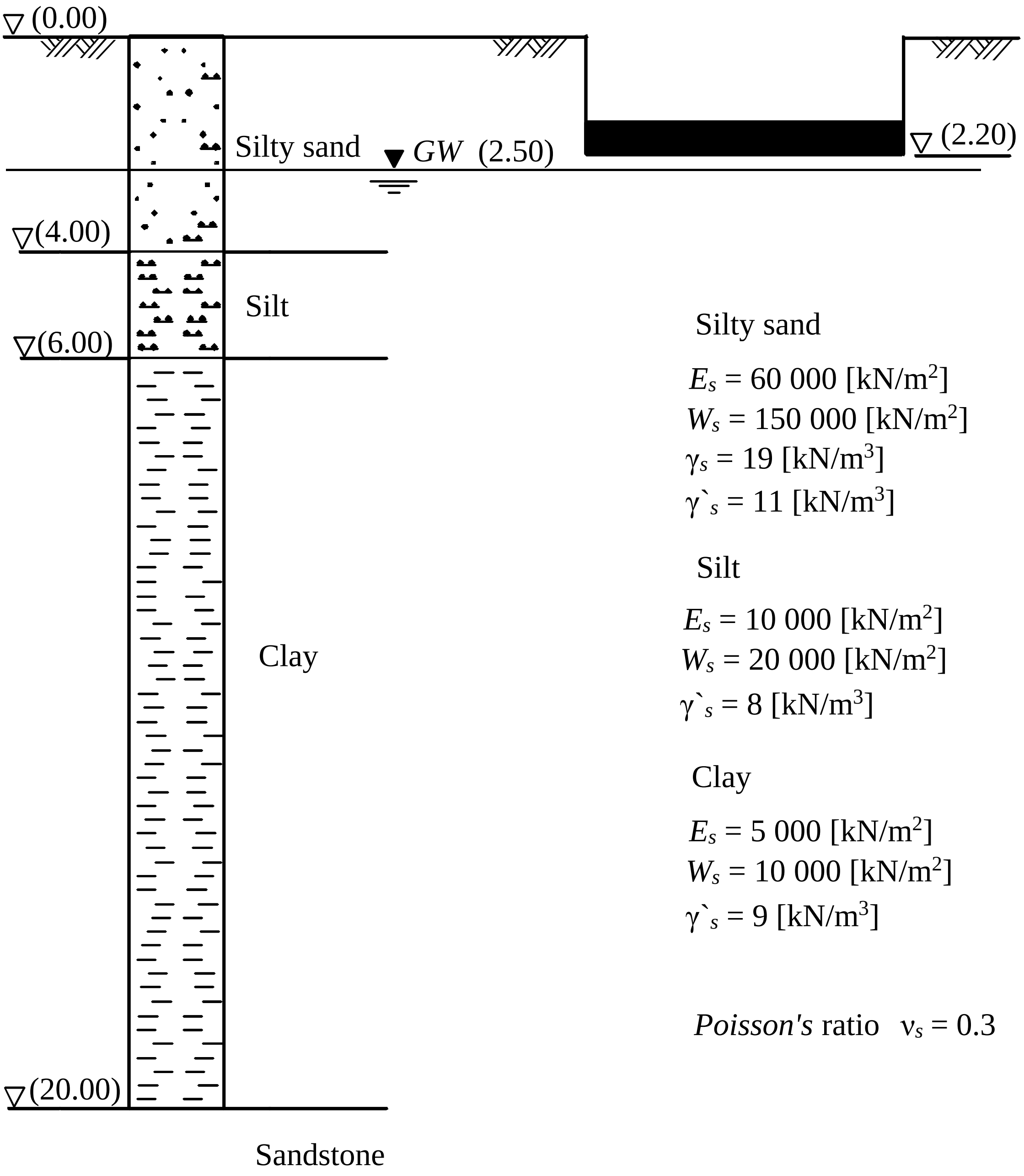
## 3 Soil properties

The rafts rest on three soil layers consisting of silty sand, silt and clay, respectively. A rigid base of sandstone is found under the clay layer. Figure 50 shows soil layers under rafts while Table 53 shows the soil parameters. *Poisson's* ratio is constant for all soil layers. The effect of reloading of the soil and limit depth of the soil layers are taken into account. The general soil parameters are:

| | | |
|---|---|---|
| *Poisson's* ratio | $\nu_s$ | = 0.30 [-] |
| Level of foundation depth under the ground surface | $d_f$ | = 2.50 [m] |
| Level of water table under the ground surface | $GW$ | = 2.20 [m] |

Table 53 Soil properties

| Layer No. I | Type of soil | Depth of layer under the ground surface $z$ [m] | Modulus of compressibility of the soil for | | Unit weight above ground water $\gamma_s$ [kN/ m$^3$] | Unit weight under ground water $\gamma'_s$ [kN/ m$^3$] |
|---|---|---|---|---|---|---|
| | | | Loading $E_s$ [kN/ m$^2$] | Reloading $W_s$ [kN/ m$^2$] | | |
| 1 | Silty sand | 4.00 | 60 000 | 150 000 | 19 | 11 |
| 2 | Silt | 6.00 | 10 000 | 20 000 | - | 8 |
| 3 | Clay | 20.0 | 5 000 | 10 000 | - | 9 |



Silty sand

$E_s$ = 60 000 [kN/m$^2$]
$W_s$ = 150 000 [kN/m$^2$]
$\gamma_s$ = 19 [kN/m$^3$]
$\gamma`_s$ = 11 [kN/m$^3$]

Silt

$E_s$ = 10 000 [kN/m$^2$]
$W_s$ = 20 000 [kN/m$^2$]
$\gamma`_s$ = 8 [kN/m$^3$]

Clay

$E_s$ = 5 000 [kN/m$^2$]
$W_s$ = 10 000 [kN/m$^2$]
$\gamma`_s$ = 9 [kN/m$^3$]

*Poisson's* ratio $\nu_s$ = 0.3

Figure 50 Soil layers and soil parameters under rafts

# 4 Analysis of the raft

## 4.1 Modeling of ribs

For modeling of ribs, different possibilities can be applied as follows:

i) The raft is analyzed first separately by considering the ribs as non-displaceable or elastic line supports. Then, the obtained support reactions apply to equivalent girders. This mathematical model supposes that the rib has more significant stiffness than that of the raft. In this case, a linear contact pressure under the raft may be assumed in the analysis (Conventional method 1), where the interaction between the raft and the subsoil is not taken into account

ii) Using a combination of two types of finite elements representing the system of a ribbed raft. The raft is represented by plate bending elements according to the two-dimensional nature of the raft. Beam elements are used to represent the rib action along the raft

iii) Using a thicker line of plate elements representing the rib action along the raft. Then, for design of the rib, the required internal forces are determined from the plate element results. This model is reasonable for a wide rib

iv) Using a three-dimensional shell model of block elements with six degrees of freedom at each node to represent the rib and raft together. This model gives an exact representation of the rib behavior but it is complicated

In this example the analysis of the ribbed raft is carried out using a combination of plate and beam elements. Figure 51 shows FE-Nets of flat and ribbed rafts. Each raft is subdivided into 312 plate elements. For the ribbed raft, the ribs are considered through inserting additional 138 beam elements along the location of the ribs on the FE-Net.

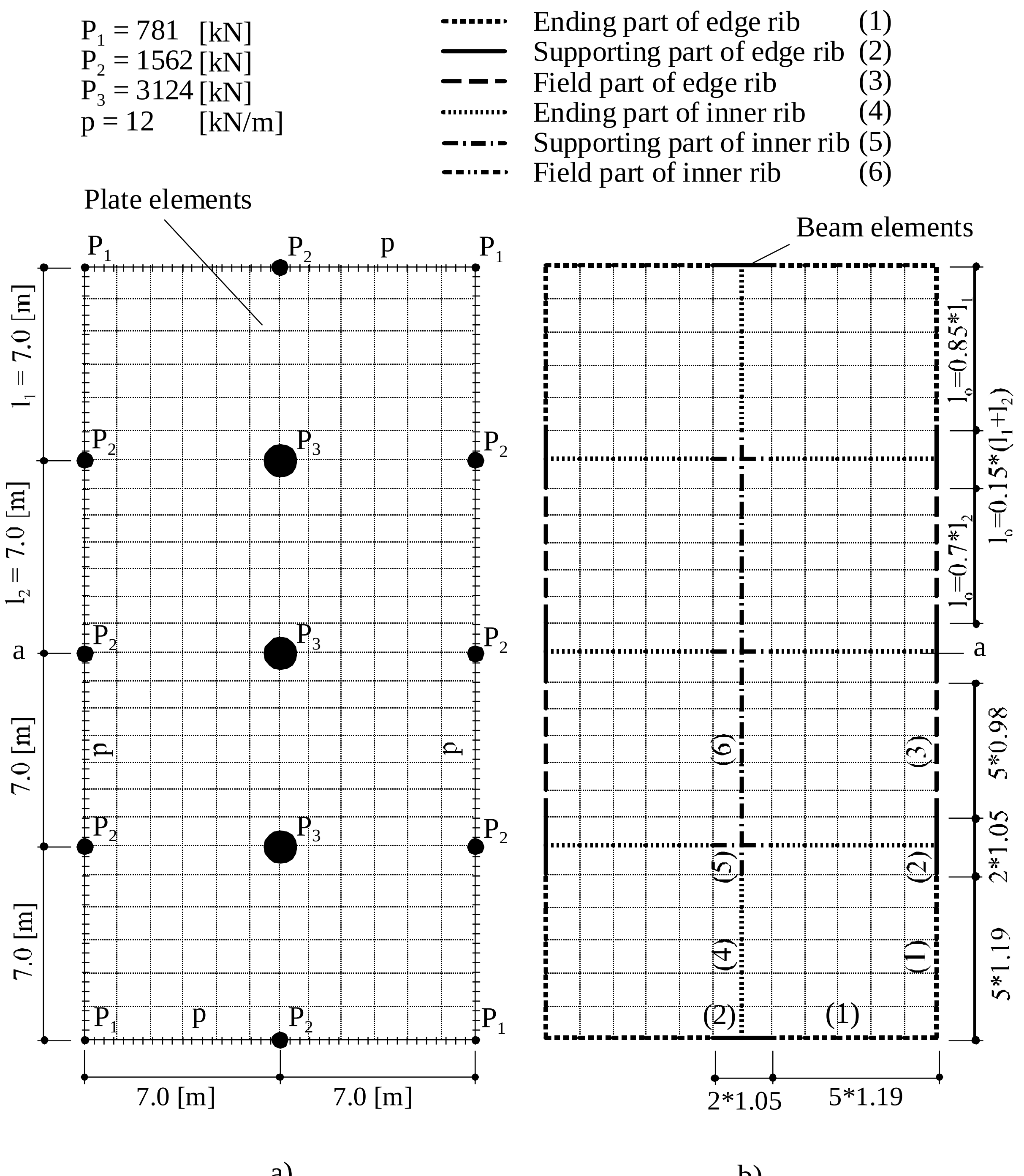


Figure 51 a) Flat raft with loads, dimensions and FE-Net
b) Ribbed raft, arrangement of beam elements, dimensions and FE-Net

## 4.2 Determination of replacement rib height $h_{Ers}$

To simulate the rib stiffness on the FE-Net by using additional beam elements, the actual properties of the beam elements must be determined. The stiffness of the rib can be obtained through a replacement beam arranged in the center plane of the plate. The dimensions of the replacement beam can be taken as in DIN 1075 or EC 2. This can be carried out by determining firstly the moment of inertia for the effective section of the rib $I_{pb}$ that contains two parts, flange and web (Figure 52). The rib section may be L-section or inverted T-section. Then, the replacement height of the web $h_{Ers}$ can be determined by equating the section of inertia $I_{pb}$ to two equivalent moments of inertia. The first moment of inertia $I_p$ corresponds a rectangular flange of dimensions $b_{eff}$ and $h_f$ while the second moment of inertia $I$ corresponds a rectangular web of dimensions $b_w$ and $h_{Ers}$. The replacement height of the web $h_{Ers}$ must be higher than the sum of slab thickness $h_f$ and clear height of the rib $h_w$. In the finite element model of the ribbed raft, the rib is represented by beam element that has the property of $b_w$ and $h_{Ers}$ while the flange is already included in the plate finite element.

According to EC 2 the rib is defined by different stiffness distribution along its length, depending on the points of zero moment at the rib, where the effective flange width of the rib depends on the position of this point. This stiffness can be determined approximately independent of the load geometry at different spans. Guidelines for calculating effective spans $l_o$ and flange widths $b_{eff}$ are given in Figure 52 and Figure 53, while Table 54 shows effective spans and flange widths of ribs at different rib parts for the raft.

Table 54 Effective span and flange width of the rib

| Rib part | Effective rib span $l_o$ [m] | Effective flange width $b_{eff}$ [m] | |
|---|---|---|---|
| | | Edge rib $b_{eff} = b_w + l_o/10$ | Inner rib $b_{eff} = b_w + l_o/5$ |
| Ending part | $0.85\ l_1 = 5.95$ | 0.895 | 1.49 |
| Supporting part | $0.15\ (l_1 + l_2) = 2.1$ | 0.51 | 0.72 |
| Field part | $0.7\ l_2 = 4.9$ | 0.79 | 1.28 |

where in Table 54:

| | |
|---|---|
| $l_1 = l_2 = 7.0$ [m] | Rib span |
| $b_w = 0.30$ [m] | Width of rib |

Figure 54 and Figure 55 show the moment of inertia ratios $r = I_{pb}/\ I$ at different clear heights $h_w$. From these figures, it can be concluded that the small clear height $h_w$ has a great influence on the ratio $r$. The replacement heights $h_{Ers}$ for different clear heights $h_w$ are plotted as curves in Figure 56 and Figure 57. These curves indicate that the maximum replacement height occurs when the clear height $h_w$ is about 0.75 – 0.80 [m]. At this clear height, the dimensions of the rib are considered optimal.

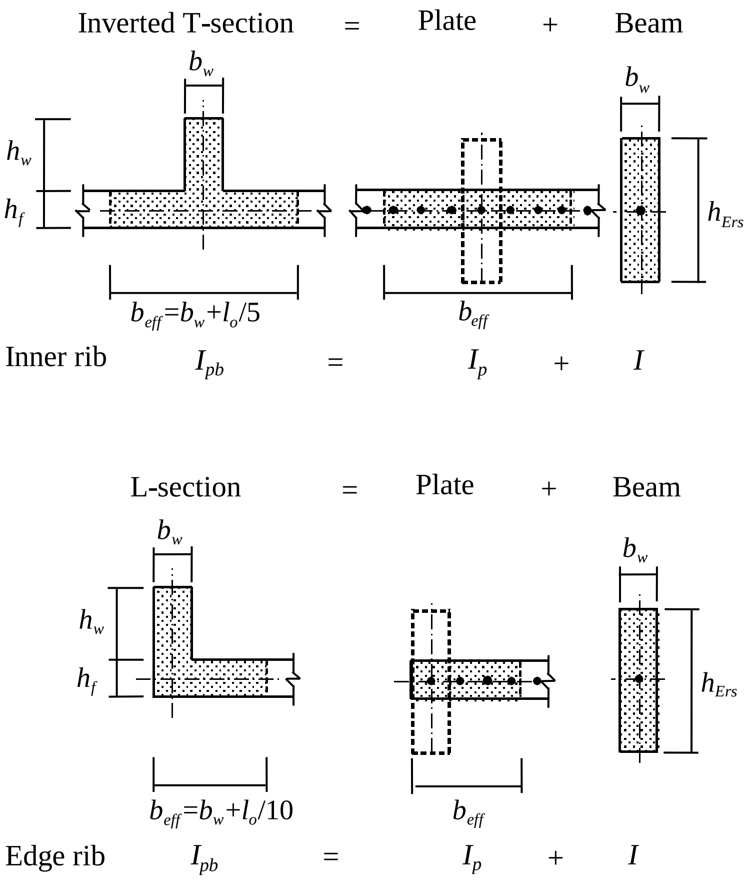


Figure 52 Determination of replacement height $h_{Ers}$

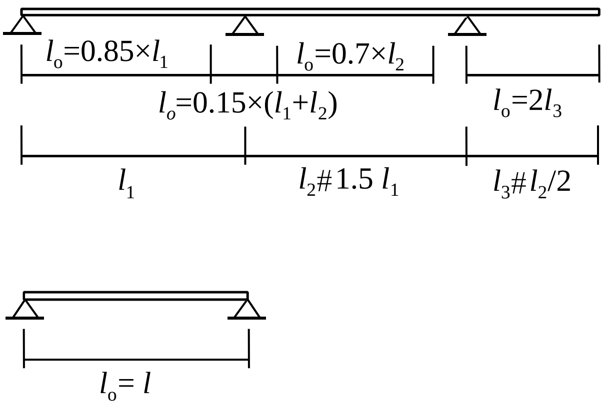


Figure 53 Definition of effective span $l_o$

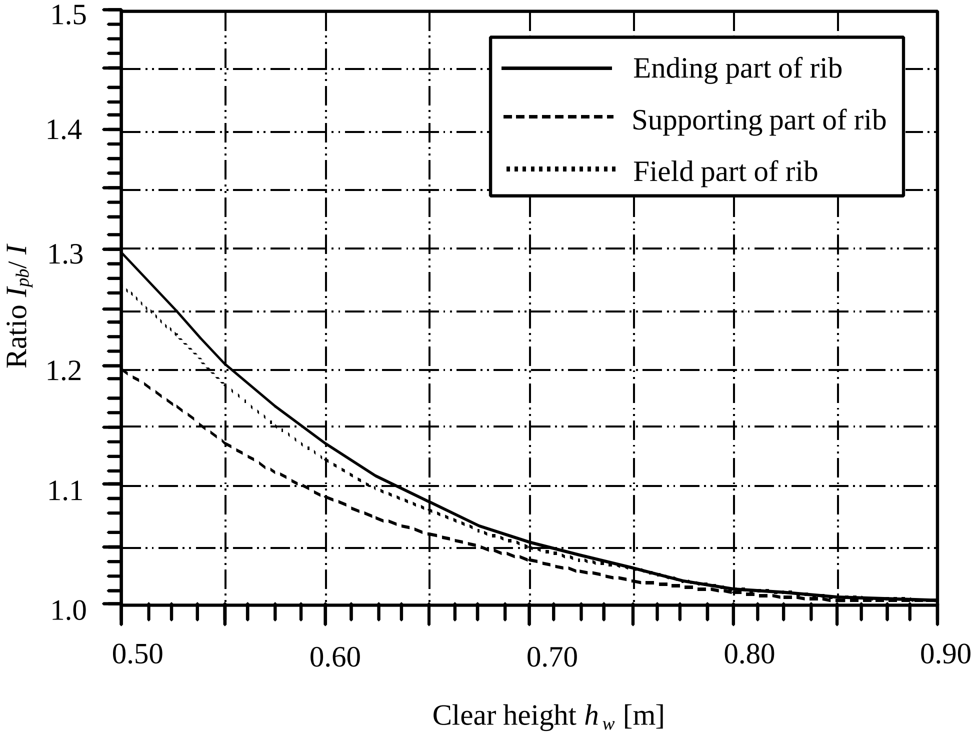


Figure 54 Moment of inertia ratio $r = I_{pb}/ I$ for edge ribs

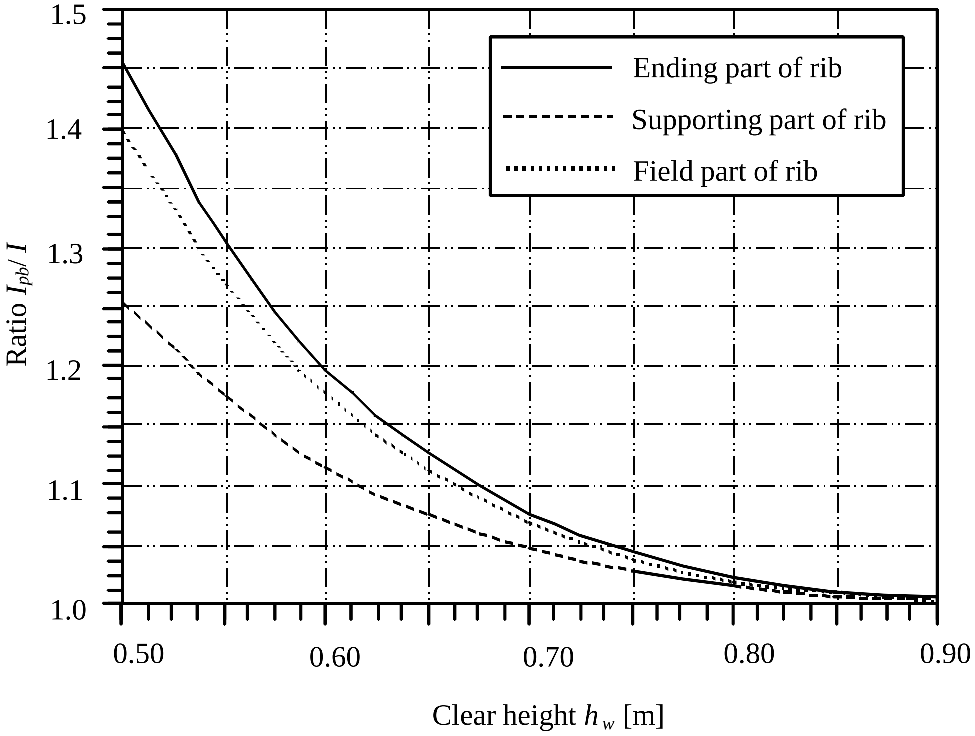


Figure 55 Moment of inertia ratio $r = I_{pb}/ I$ for inner ribs

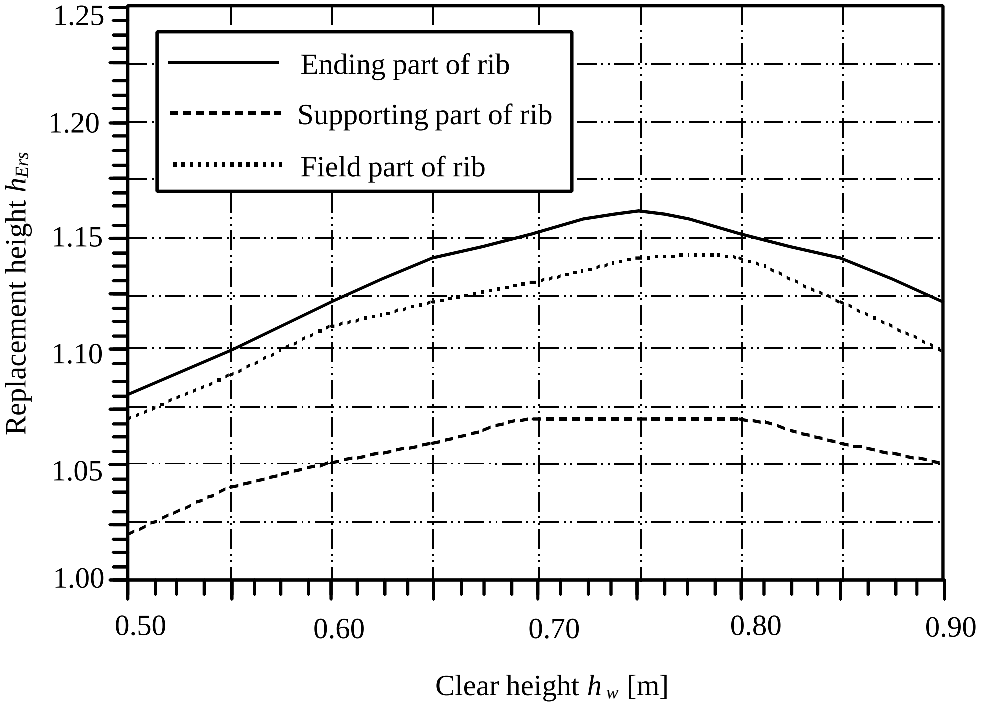


Figure 56 Replacement height $h_{Ers}$ for edge ribs

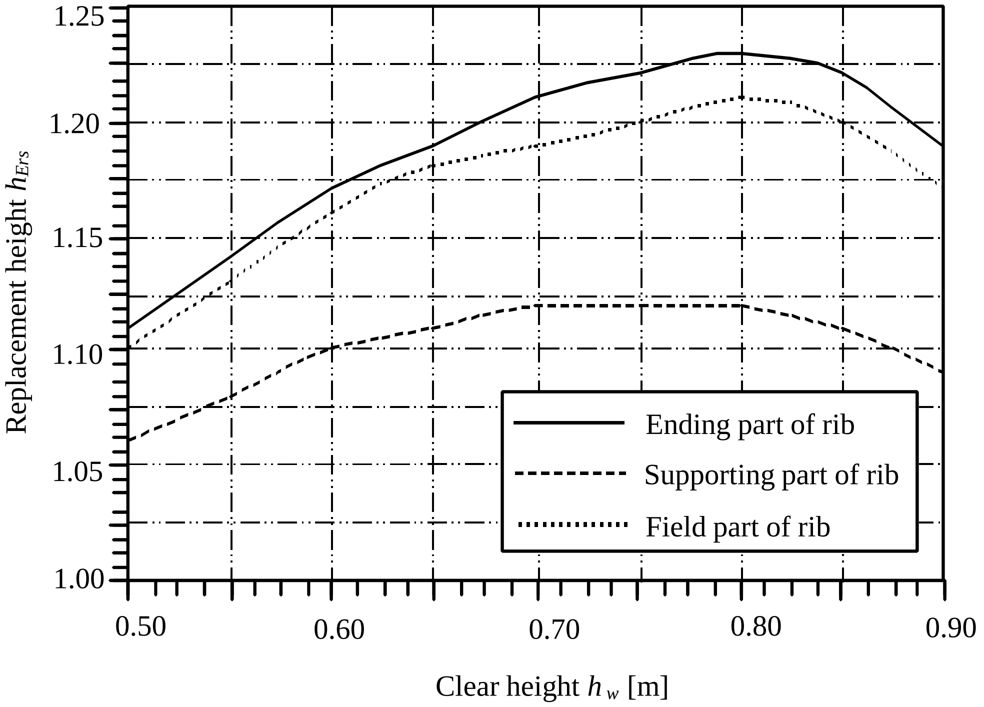


Figure 57 Replacement height $h_{Ers}$ for inner ribs

## 4.3 Optimal thickness

The optimal thickness is designed to be the minimum thickness of the raft for which the concrete section and tensile reinforcement are enough to resist the flexure moments without compressive reinforcement. The optimal thickness is designed according to EC 2 for the following parameters:

### Material

| | |
|---|---|
| Concrete grade | C 30/37 |
| Steel grade | BSt 500 |
| Characteristic compressive cylinder strength of concrete | $f_{ck} = 30$ [MN/ m$^2$] |
| Characteristic tensile yield strength of reinforcement | $f_{yk} = 500$ [MN/ m$^2$] |
| Partial safety factor for concrete strength | $\gamma_c = 1.5$ |
| Design concrete compressive strength | $f_{cd} = f_{ck}/\ \gamma_c = 30/\ 1.5 = 20$ [MN/ m$^2$] |
| Partial safety factor for steel strength | $\gamma_s = 1.15$ |
| Design tensile yield strength of reinforcing steel | $f_{yd} = f_{yk}/\ \gamma_s = 500/\ 1.15 = 435$ [MN/ m$^2$] |

### Geometry

| | |
|---|---|
| Width of the section to be designed | $b = 1.0$ [m] |
| Concrete cover + 1/2 bar diameter | $c = 5.0$ [cm] |

### Factored moment

The maximum moment $m_{max}$ for the raft is obtained at different raft thicknesses $t$ for flat raft and slab thicknesses $h_f$ for ribbed raft. As soil layers represent the subsoil under the rafts, one of the methods for Continuum model may be used. The considered rafts and system of loads will lead to appearing a negative contact pressure, if method 6 or 7 is used. Therefore, the modification of modulus of subgrade reaction by iteration (method 4) with sufficient accuracy $\varepsilon = 0.002$ [m] is used in the analyses. It is found that the maximum moment $m_{max}$ for the flat raft occurs always at its center while for the ribbed raft occurs at different places depending on the slab thickness.

| | | |
|---|---|---|
| Total load factor for both dead and live loads | $\gamma$ | $= 1.5$ |
| Factored moment | $M_{sd}$ | $= \gamma m_{max}$ |

### Check for section capacity

The limiting value of the ratio $x/\ d$ is $\xi_{lim} = 0.45$ for $f_{ck}$ # 35 [MN/ m$^2$]

The normalized concrete moment capacity $\mu_{sd,\ \mathrm{lim}}$ as a singly reinforced section is

$$\mu_{sd,\ lim} = 0.8\xi_{\mathrm{lim}}(1 - 0.4\xi_{\mathrm{lim}})$$

$$\mu_{sd,\ lim} = 0.8 \times 0.45(1 - 0.4 \times 0.45) = 0.295$$

The sustained moment $M_a$ for singly reinforced section will be obtained from

$$\mu_{sd,\ lim} = \frac{M_a}{bd^2(0.85 f_{cd})}$$

$$M_a = \mu_{sd,\,\mathrm{lim}}\, bd^2(0.85\, f_{cd}) = 0.295\times1.0\times d^2\times0.85\times20$$

$$M_a = 5.015\, d^2$$

where for flat raft:
$d$ = $t$ - 5 [cm] cover
$t$ = raft thickness for flat raft

and for ribbed raft:
$d$ = $h_f$ - 5 [cm] cover
$h_f$ = slab thickness for ribbed raft

The factored moment $M_{sd}$ and the sustained moment $M_a$ for both flat and ribbed rafts are calculated at different thicknesses and plotted in Figure 58 and Figure 59. The minimum thickness is obtained from the condition $M_{sd} = M_a$. From Figure 58 and Figure 59 the minimum thickness for the flat raft is $t$ = 0.58 [m] while for the ribbed raft is $h_f$ = 0.24 [m]. Therefore, the optimal thickness for the flat raft is chosen to be $t$ = 0.60 [m] while for the ribbed raft is chosen to be $h_f$ = 0.25 [m]. Table 55 shows a comparison between flat and ribbed rafts, which indicates that ribbed raft lead to less concrete volume and weight than those of flat raft by 44 [%].

Table 55 Comparison between flat and ribbed rafts

| Cases | Concrete volume [m$^3$] | Concrete weight [kN] | Average contact pressure $\sigma_o$ [kN/ m$^2$] |
|---|---|---|---|
| Flat raft | 243 | 6070 | 81 |
| Ribbed raft | 135 | 3384 | 75 |
| Difference [%] | 44 | 44 | 7 |

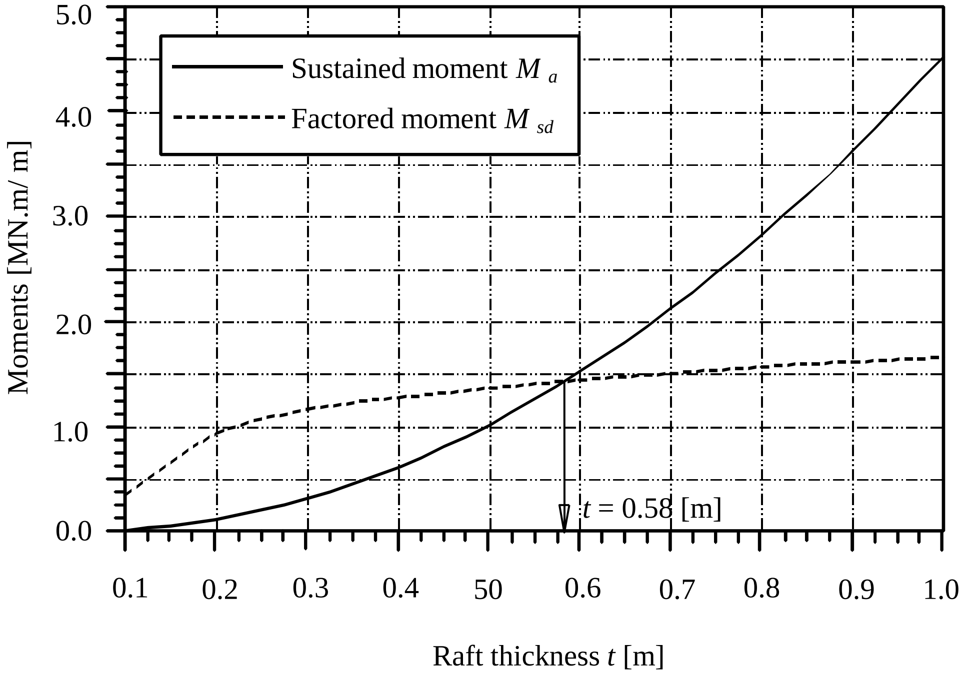


Figure 58 Determination of optimal raft thickness $t$ for flat raft

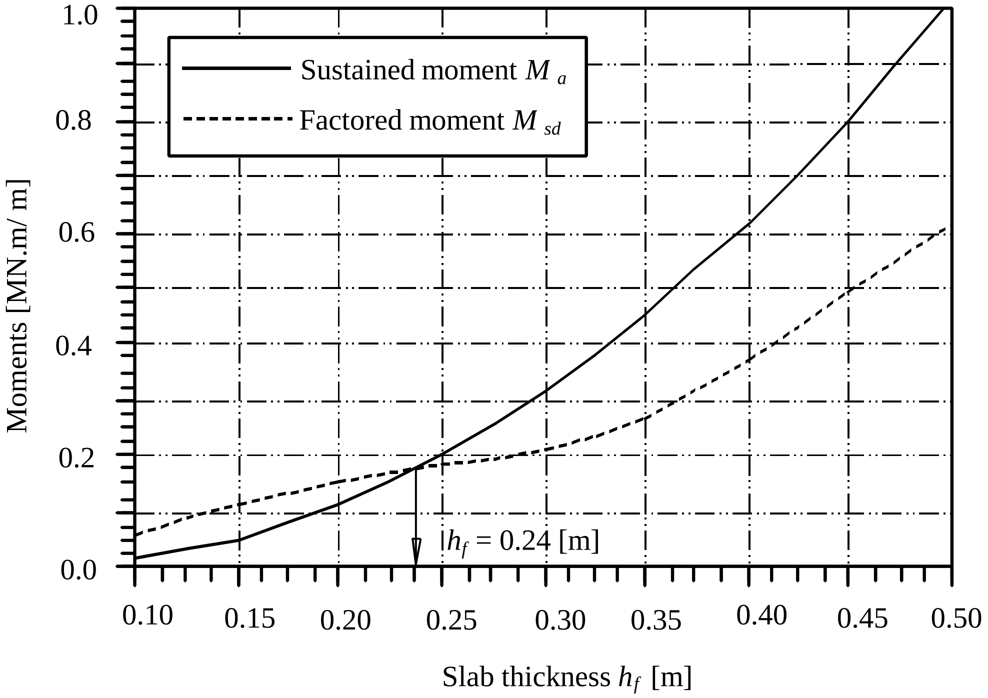


Figure 59 Determination of optimal slab thickness $h_f$ for ribbed raft

Details of rib properties concerning ribbed raft are shown in Table 56.

Table 56 Properties of ribs for slab thickness $h_f$ = 0.25 [m] and clear height $h_w$ = 0.75[m]

| Rib part | | Moment of inertia for effective rib section $I_{pb}$ [m$^4$] | Moment of inertia for rib flange $I_b$ [m$^4$] | Replacement web | | |
|---|---|---|---|---|---|---|
| | | | | Replacement height $h_{Ers}$ [m] | Moment of inertia $I$ [m$^4$] | Torsional inertia $J$ [m$^4$] |
| Edge rib | Ending | 0.0398 | 0.0386 | 1.16 | 0.0390 | 0.0087 |
| | Supporting | 0.0316 | 0.0309 | 1.07 | 0.0306 | 0.0079 |
| | Field | 0.0379 | 0.0368 | 1.14 | 0.0370 | 0.0086 |
| Inner rib | Ending | 0.0476 | 0.0456 | 1.22 | 0.0454 | 0.0093 |
| | Supporting | 0.0365 | 0.0355 | 1.12 | 0.0351 | 0.0084 |
| | Field | 0.0452 | 0.0436 | 1.20 | 0.0432 | 0.0091 |

where:

Moment of Inertia for rib $I = b_w\, h_{Ers}{}^3 / 12$

Torsional Inertia for rib $J = h_{Ers} b_w{}^3 \left(\frac{1}{3} - 0.21 \frac{b_w}{h_{Ers}}\right)\left(1 - \frac{b_w{}^4}{12 h_{Ers}{}^4}\right)$

## 4.4 Determination of the limit depth $t_s$

The level of the soil under the raft in which no settlement occurs or the expected settlement will be very small, where can be ignored, is determined as a limit depth of the soil. The limit depth in this example is chosen to be the level of which the stress due to the raft $\sigma_E$ reaches the ratio $\xi$ = 0.2 of the initial vertical stress $\sigma_V$. The stress in the soil $\sigma_E$ is determined at the characteristic point *c* of the rectangular foundation. The stress $\sigma_E$ is due to the average stress from the raft at the surface $\sigma_O$ = 81 [kN/ m$^2$] for flat raft and $\sigma_O$ = 75 [kN/ m$^2$] for ribbed raft. At the characteristic point, from the definition of *Graßhoff* (1955), the settlement if the raft is full rigid will be identical with that if the raft is full flexible. The characteristic point *c* takes coordinates *x* = 0.87 × *L* = 12.18 [m] and *y* = 0.13 × *B* = 3.64 [m] as shown in Figure 59. The results of the limit depth calculation are plotted in a diagram as shown in Figure 59. The limit depth is $t_s$ = 13.55 [m] for flat raft and $t_s$ = 12.93 [m] for ribbed raft under the ground surface.