Example 6: Design of trapezoidal footing

1 Description of the problem

In the primary design of footings or rafts, it is generally assumed that the contact pressure distribution is planar, whatever the type of model used in the analysis of the footing. Therefore, to achieve a desirable uniform contact stress distribution beneath the footing it is necessary to arrange the center of area of the footing directly beneath the center of gravity of the external loads. This may lead to irregular-shaped footing. If equal column loads are symmetrically disposed about the center of the footing, the contact pressure distribution will be uniform. If this not the case, a theoretically uniform contact pressure distribution should be achieve. In order to do that, the footing can be extended so that the center of area of the footing coincides with the center of gravity of the external loads. This is easy to be done by rectangular footing.

A special case of footings is the trapezoidal footing, which may be used to carry two columns of unequal loads when distance outside the column of the heaviest load is limited. In such case, using a rectangular footing may lead to the resultant of loads which do not fall at the middle length of the footing. To overcome this difficulty, a trapezoidal footing is used in such a way that the center of gravity of the footing lies under the resultant of the loads. Correspondingly, the distribution of contact pressure will be uniform.

As a design example for trapezoidal footing, consider the trapezoidal combined footing of 0.60 [m] thickness shown in Figure 80. The footing is supported to two columns C1 and C2 spaced at 4.80 [m] apart. Due to the site conditions, the projections of the footing beyond the centers of columns C1 and C2 are limited to 0.90 [m] and 1.30 [m], respectively. Column C1 is 0.50 [m] × 0.50 [m], reinforced by 8 Φ 16 [mm] and carries a load of 1200 [kN]. Column C2 is 0.60 [m] × 0.60 [m], reinforced by 12 Φ 19 [mm] and carries a load of 2000 [kN]. The allowable net soil pressure is $(q_{net})_{\text{all}}$ = 240 [kN/m^2]. The subsoil model used in the analysis of the footing is represented by isolated springs, which have a modulus of subgrade reaction of k_s = 50 000 [kN/m^3]. A thin plain concrete of thickness 0.15 [m] is chosen under the footing and is unconsidered in any calculations.

2 Footing section and material

The footing material and section are supposed to have the following parameters:

2.1 Section properties

The material of rafts is reinforcement concrete that has the following parameters:

Width of the section to be designed	b = 1.0	[m]
Section thickness	t = 0.60	[m]
Concrete cover + 1/2 bar diameter	c = 5	[cm]
Effective depth of the section	$d = t - c$ = 0.55	[m]
Steel bar diameter	Φ = 25	[mm]

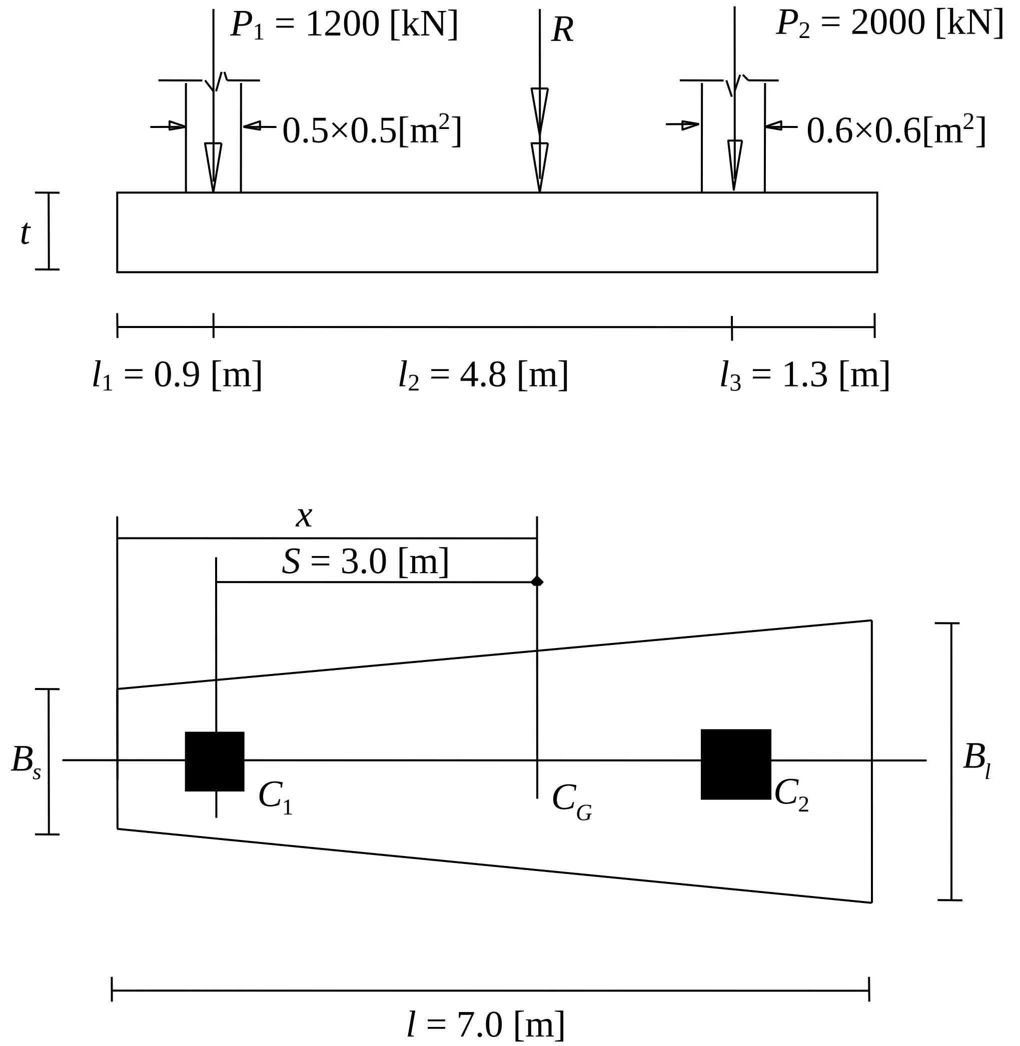


Figure 80 Combined trapezoidal footing

2.2 Material properties

Concrete grade according to ECP	C 250		
Steel grade according to ECP	S 36/52		
Compressive stress of concrete	f_c = 95	[kg/cm^2] = 9.5	[MN/m^2]
Tensile stress of steel	f_s = 2000	[kg/cm^2] = 200	[MN/m^2]
Young's modulus of concrete	$E_b = 3 \times 10^7$	[kN/m^2] = 30000	[MN/m^2]
Poisson's ratioof concrete	ν_b = 0.20	[-]	
Unit weight of concrete	γ_b = 0.0	[kN/m^3]	

Unit weight of concrete is chosen $\gamma_b = 0.0$ to neglect the self weight of the footing.

3 Analysis of the footing

3.1 Determination of footing sides B_s and B_l

The primary design required to establish the area of footing so that the center of area of the footing coincides with the center of gravity of the resultant. This will be conducted as follows:

Resultant of loads R is given by:

$$R = P_1 + P_2 = 1200 + 2000 = 3200 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{(all)net}} = \frac{3200}{240} = 13.33 [\text{m}^2]$$

Referring to Figure 80, area of footing A_f is given by:

$$A_f = \frac{l}{2}\left(B_s + B_l\right)$$

$$13.33 = \frac{7}{2}\left(B_s + B_l\right)$$

Simplifying,

$$B_s + B_l = 3.8 \text{ [m]} \qquad \text{(i)}$$

Taking the moment of the column loads about the center of the column C1, the distance S between the point of application of the resultant and the center of column C1 is obtained from:

$$S \times R = P_2 \times l_2$$

$$S \times 3200 = 2000 \times 4.8$$

$$S = 3.0 \text{ [m]}$$

Hence, the point of application of the resultant is also the centroid of the footing area. Therefore, it can be shown from the geometry of the footing that the distance x from the small side B_s to the center of area is given by

$$x = \frac{l}{3}\frac{B_s + 2B_l}{B_s + B_l}$$

$$l_1 + S = \frac{l}{3}\frac{B_s + 2B_l}{B_s + B_l}$$

$$0.9 + 3.0 = \frac{7}{3}\frac{B_s + 2B_l}{B_s + B_l}$$

Simplifying,

$$2.04B_s - B_l = 0 \qquad \text{(ii)}$$

Solving Equation (i) and (ii) yields the required dimensions of B_s and B_l as follows:

$$B_s = 1.25 \text{ [m] and } B_l = 2.56 \text{ [m]}$$

Chosen dimensions of B_s and B_l are:

$$B_s = 1.30 \text{ [m] and } B_l = 2.60 \text{ [m]}$$

3.2 Finite element analysis

The footing is subdivided into 92 quadrature, rectangular and triangular elements to fit the exact area of the footing as shown in Figure 81.

If a point load represents the column load on the mesh of fine finite elements, the moment under the column will be higher than the real moment. Therefore, the column load is distributed at the centerline of the footing on an area of $(a + d)^2$ as shown in Figure 81 through activation the option of distribution column load in *ELPLA*. Figure 82 shows the calculated contact pressure q [kN/m^2], while Figure 83 shows the moment m_x [kN.m/m] at the critical section *I-I* of the footing. Figure 84 shows the distribution of the moment m_y [kN.m/m] in the plan. For ECP codes, the footing is designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained.

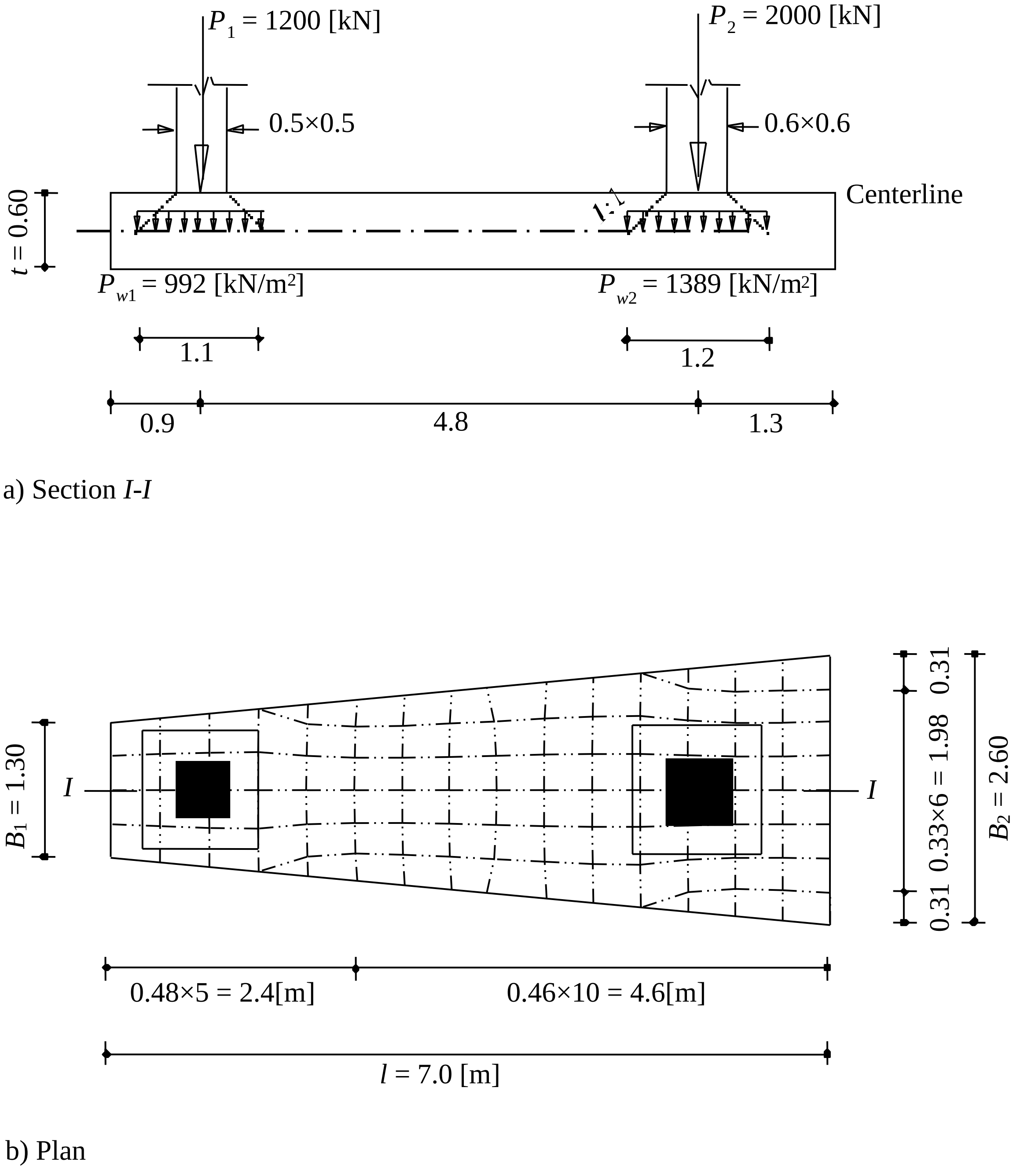
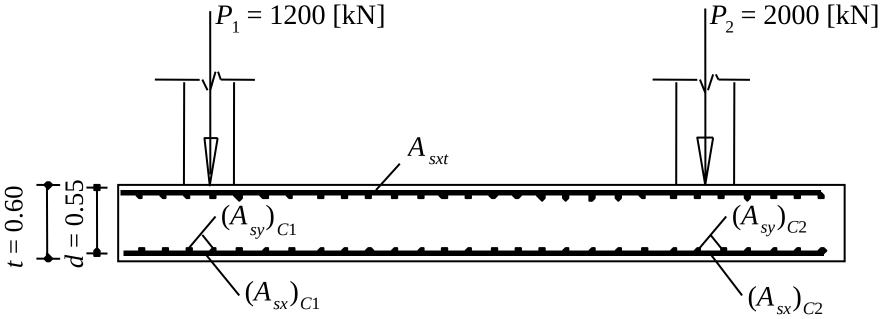


Figure 81 FE-Net and distribution of column loads through the footing



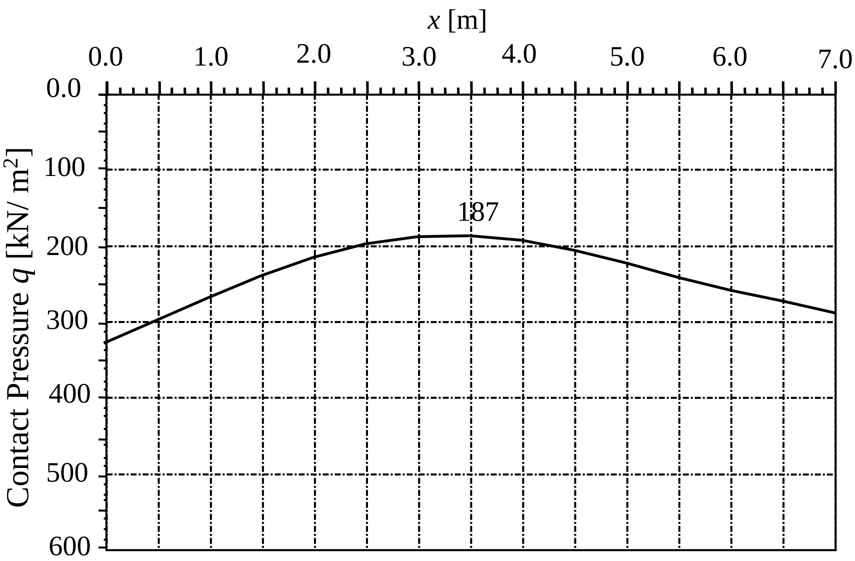


Figure 82 Contact pressure q [KN/ m^2] at section *I-I*

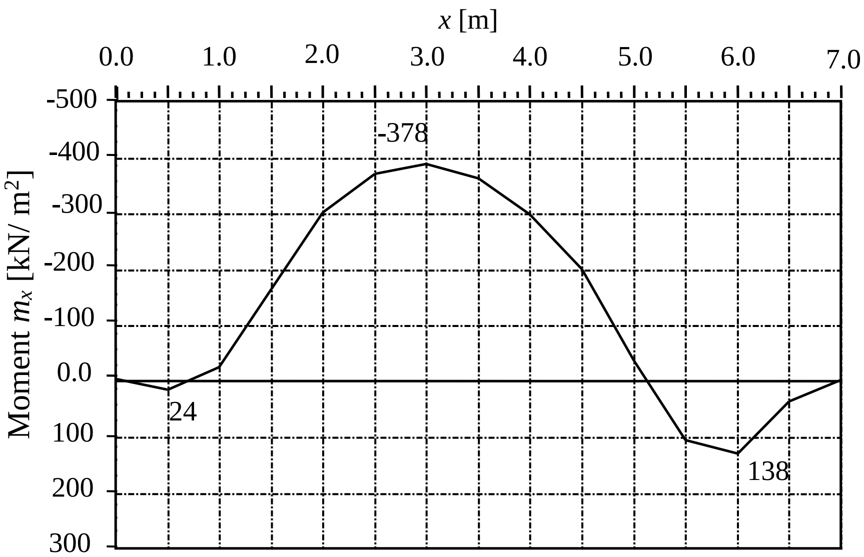


Figure 83 Moment m_x [KN.m/ m] at section *I-I*

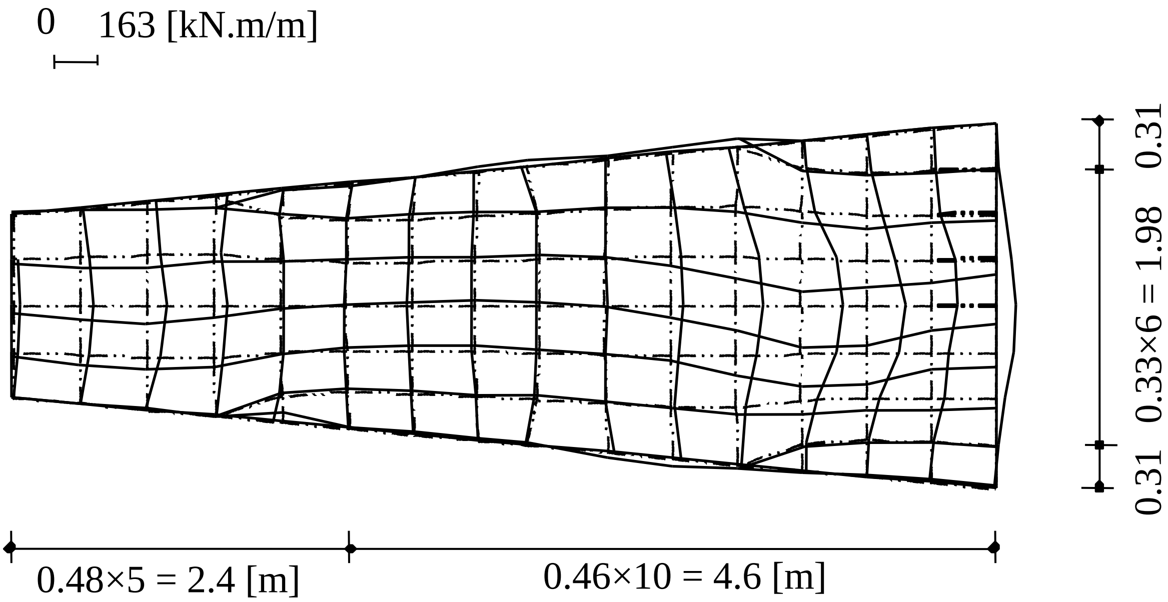


Figure 84 Distribution of the moment m_y [kN.m/ m] in the plan

4 Design for ECP (working stress method)

Material

Concrete grade	*C* 250
Steel grade	*S* 36/52
Compressive stress of concrete	f_c = 95 [kg/ cm^2] = 9.5 [MN/ m^2]
Tensile stress of steel	f_s = 2000 [kg/ cm^2] = 200 [MN/ m^2]

Maximum moment

Maximum moment per meter at critical section obtained from analysis

M = 378 [kN.m] = 0.4 [MN.m]

Geometry

Effective depth of the section	d = 0.55 [m]
Width of the section to be designed	b = 1.0 [m]

Determination of depth required to resist moment d_m

From Table 68 for f_c = 9.5 [MN/ m^2] and f_s = 200 [MN/ m^2], the coefficient k_1 to obtain the section depth at balanced condition is k_1 = 0.766, while the coefficient k_2 [MN/ m^2] to obtain the tensile reinforcement for singly reinforced section is k_2 = 172 [MN/ m^2].

The maximum depth d_m as a singly reinforced section is given by

$$d_m = k_1 \sqrt{\frac{M}{b}}$$

$$d_m = 0.766\sqrt{\frac{0.40}{1.0}} = 0.48[\text{m}]$$

Take d = 0.55 [m] > d_m = 0.48 [m], then the section is designed as singly reinforced section.

Check for punching shear

The critical punching shear section on a perimeter at a distance $d/2$ = 0.275 [m] from the face of the column is shown in Figure 85. The check for punching shear under columns C1 and C2 is shown in Table 68.

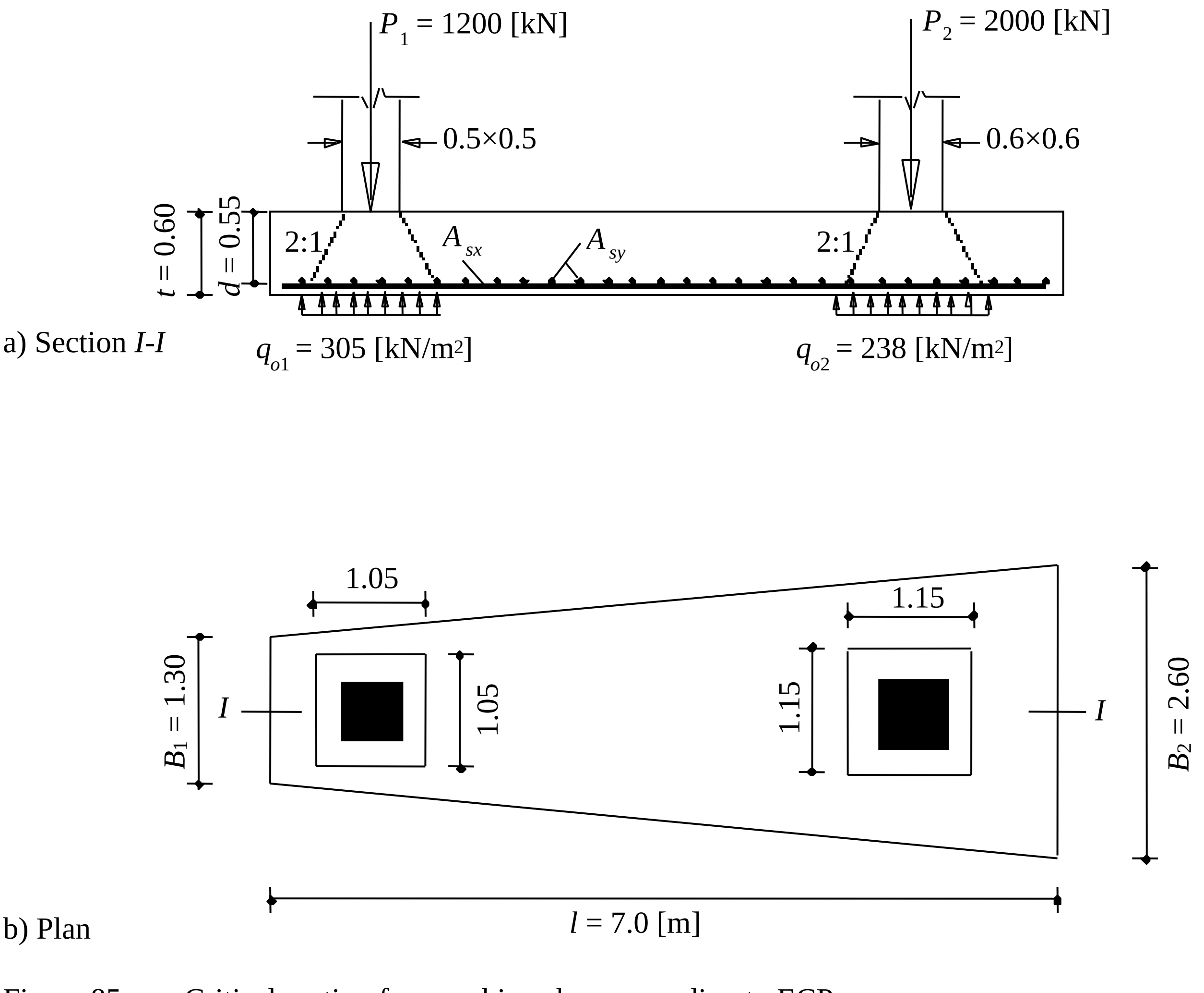


Figure 85 Critical section for punching shear according to ECP

Table 68 Check for punching shear

Load, stress and geometry	Column C1	Column C2
Column load P [MN]	1.2	2.0
Contact pressure q_o [MN/ m^2]	0.273	0.248
Column sides $a \times b$ [m^2]	0.5 × 0.5	0.6 × 0.6
Footing thickness d [m]	0.55	0.55
Critical perimeter $b_o = 4\ (a + b)$ [m]	4.2	4.6
Critical area $A_p = (a + d)^2$ [m^2]	1.1025	1.3225
Punching load $Q_p = P - q_o.A_p$ [MN]	0.9	1.66
Punching shear stress $q_p = Q_p/\ (b_o.d)$ [MN/ m^2]	0.386	0.390

The allowable concrete punching strength q_{pall} [MN/ m^2] is given by

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp}, \leq q_{cp}$$

$$q_{pall} = (0.5 + 1.0)\,0.9, \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/ m}^2\text{]}$$

For both columns $q_{pall} > q_p$, the footing section is safe for punching shear.

Determination of tension reinforcement

Minimum area of steel reinforcement $A_{s.min}$ = 0.15 [%], A_c = 0.0015 × 60 × 100 = 9 [cm^2/ m].
Take $A_{s.min}$ = 5 Φ 16/ m = 10.1 [cm^2/ m].

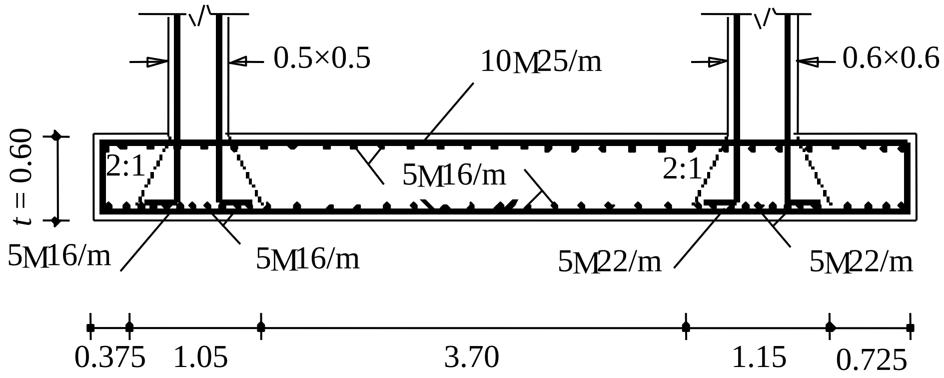
The determination of the required area of steel reinforcement in both x- and y-directions is shown in Table 69 and Table 70. The details of reinforcement in plan and section *a-a* through the footing are shown in Figure 86.

Table 69 Determination of tension reinforcement for *x*-direction

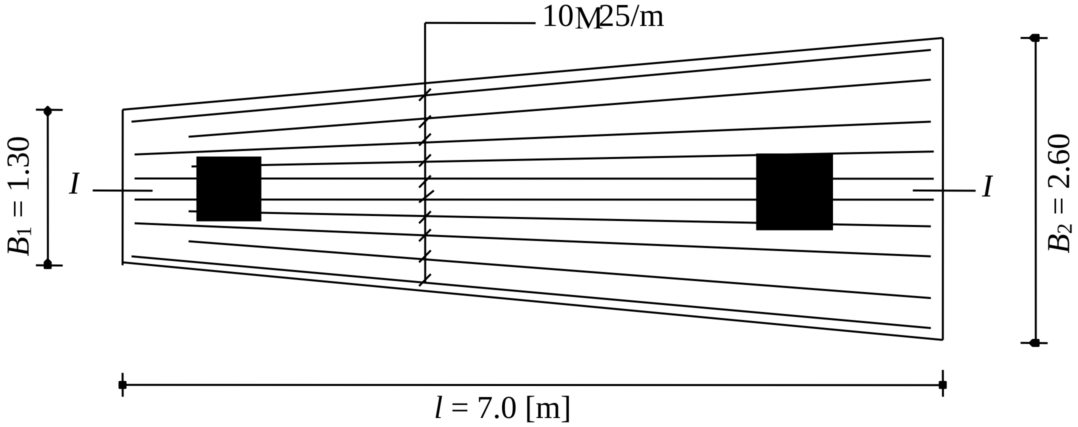
Position	Moment M [MN.m/ m]	Calculated A_s $A_s = M/ (k_2.d)$ [cm^2/ m]	Chosen reinforcement A_s
A_{sxt}	0.378	39.51	10 Φ 25/ m
$(A_{sxb})_{C1}$	0.024	0.22	5 Φ 16/ m = $A_{s.min}$
$(A_{sxb})_{C2}$	0.138	14.58	5 Φ 22/ m

Table 70 Determination of tension reinforcement for *y*-direction

Position	Moment M [MN.m/ m]	Calculated A_s $A_s = M/ (k_2.d)$ [cm^2/ m]	Chosen reinforcement A_s
$(A_{syb})_{C1}$	0.072	6.86	5 Φ 16/ m = $A_{s.min}$
$(A_{syb})_{C2}$	0.163	16.22	5 Φ 22/ m



a) Section *I-I*



b) Plan

Figure 86 Details of reinforcement in plan and section *a-a* through the footing