Example 7: Design of a group of footings with and without tie beams

1 Description of the problem

This example shows analysis and design of a group of footings resting on an elastic foundation by two different structural systems. In the first one, the group of footings has no connections while in the second one, the group of footings is connected together by stiff tie beams considering the interaction effect among footings, tie beams and the subsoil as one unit. Finally, a comparison is carried out between the two structural systems.

It is obviously that, if there is no accurate method to determine the stress due to the interaction between the footings and tie beams, the purpose of the presence of the tie beams in this case will be only carrying the walls of the ground floor. Here it is impossible to construct the walls directly on the soil. In the other case, the presence of the tie beams is unnecessary when walls for the ground floor are not required. It is impossible in any way to depend on the tie beams for reducing the deferential settlements for footing or footing rotations without perfect knowledge about the extent of their effect in the structural analysis accurately.

The program *ELPLA* has the possibility to composite two types of finite elements in the same net. In which, the footings are represented by plate elements while the tie beams are represented by beam elements. Thus, footings and tie beams can be analyzed correctly.

Figure 87 shows a layout of columns for a multi-storey building. The columns are designed to carry five floors. The dimensions of columns, reinforcement and column loads are shown in the same Figure 87.

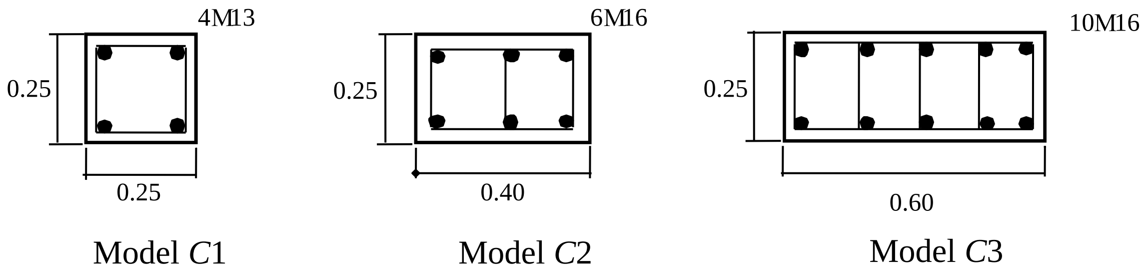
It is required to design the building footings considering property lines at the west and south sides of the building (a neighbor building). The design must be carried out twice. In the first one, the footings are designed as isolated footings without connection among them, while in the second the footings are designed as connected footings with tie beams to reduce the differential settlements among them and footing rotations.

2 Soil properties

The soil under the foundation level till the end of the boring up to 10 [m] consists of homogeneous middle sand with the following parameters:

Allowable net bearing capacity of the soil	$(q_{net})_{all}$	= 200	[kN/ m^2]
Modulus of subgrade reaction	k_s	= 40 000	[kN/ m^3]
Proposal foundation level	d_f	= 1.5	[m]

The level of the groundwater is G_w = 3.0 [m] under the ground surface. The groundwater effect is neglected in the analysis of footings, because the groundwater level is lower than the foundation level.



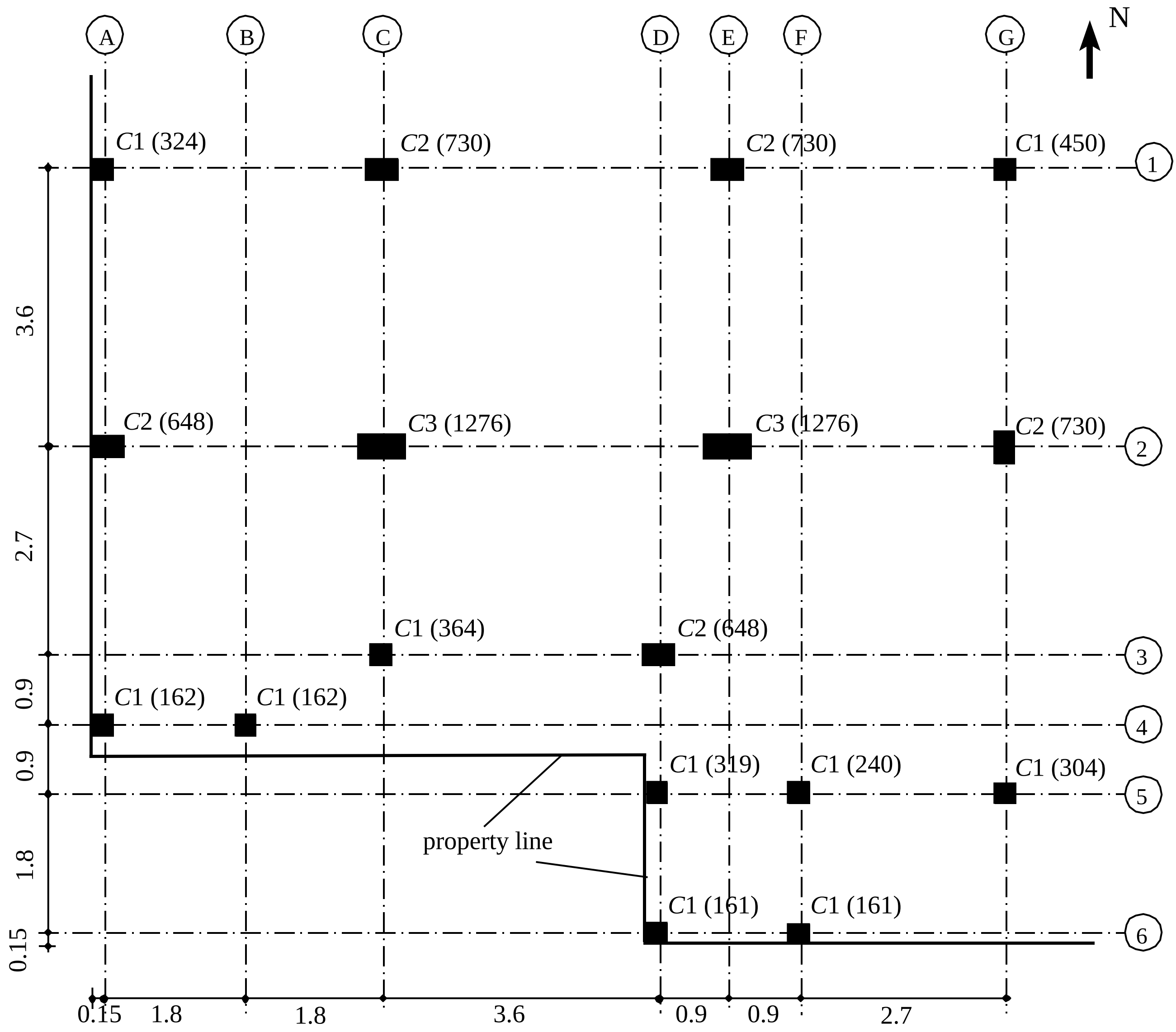


Figure 87 Layout of columns, column loads [kN] and models

3 Choice of the mathematical soil model

The system of the suggested foundation for the building is a group of footings, which have limited areas. This system of foundation doesn't require a complicated soil model, because the contact pressure between the soil and the footing in most cases will be nearly uniform, especially, if the footing area is chosen carefully so that the center of gravity of the footing area lies on the point of application of the resultant force. In this case, the choice of the simple assumption model that considers a linear contact pressure under the footing is acceptable. But the disadvantage of this model is the neglect of the interaction between the footing and the soil. In this example, a mathematical soil model, which is more accurate than the simple assumption model, is used. The mathematical model is *Winkler's* model, which represents the soil by a group of elastic springs.

4 Footing material and section properties

The design of footings and tie beams is carried out by the working method according to ECP. Footing material and section are supposed to have the following parameters:

4.1 Material properties

Concrete grade according to ECP	C 250			
Steel grade according to ECP	S 36/52			
Concrete cube strength	$f_{cu} = 250$	[kg/ cm^2]	= 25	[MN/ m^2]
Concrete cylinder strength	$f'_c = 0.8\ f_{cu}$	[-]	= 20	[MN/ m^2]
Compressive stress of concrete	$f_c = 95$	[kg/ cm^2]	= 9.5	[MN/ m^2]
Main value of shear strength	$q_{cp} = 9$	[kg/ cm^2]	= 0.9	[MN/ m^2]
Allowable shear stress of concrete	$q_c = 9$	[kg/ cm^2]	= 0.9	[MN/ m^2]
Allowable bond stress	$q_b = 12$	[kg/ cm^2]	= 1.2	[MN/ m^2]
Tensile stress of steel	$f_s = 2000$	[kg/ cm^2]	= 200	[MN/ m^2]
Reinforcement yield strength	$f_y = 3600$	[kg/ cm^2]	= 360	[MN/ m^2]
Young's modulus of concrete	$E_b = 3 \times 10^7$	[kN/ m^2]	= 30000	[MN /m^2]
Shear modulus of concrete	$G_b = 1.3 \times 10^7$	[kN/ m^2]	= 13000	[MN /m^2]
Poisson's ratio of concrete	$\nu_b = 0.15$	[-]		
Unit weight of concrete	$\gamma_b = 0.0$	[kN/ m^3]		

Unit weight of concrete is chosen $\gamma_b = 0.0$ to neglect the own weight of the footing.

4.2 Minimum section properties and reinforcement

Concrete cover + 1/2 bar diameter	$c = 5$	[cm]
Minimum steel bar diameter in footings and tie beams	$\Phi = 18$	[mm]
Minimum number of steel bars	$n = 5$	[bars]
Minimum footing thickness	$t = 0.3$	[m]
Minimum footing length	$l = 1.1$	[m]

The allowable minimum area of steel in the footings and tie beams is 0.15 [%] from the concrete section ($min\ A_s = 0.0015\ A_c$).

5 Plain concrete properties

The reinforcement concrete cannot be constructed directly on the ground. Therefore, a thin plain concrete of thickness 15 [cm] under the footings and tie beams is used. The plain concrete is not considered in any calculation because of its weakness.

6 Structural analysis and design

6.1 Footing areas

The area of each footing is determined so that the contact pressure between the footing and the soil does not exceed than the net allowable capacity of the soil $(q_{net})_{all}$ = 200 [kN/ m^2]. To avoid the footing rotation, isolated footings are chosen to be support for interior columns while combined footings are chosen to be support for exterior columns. It must be considered that the point of application of the force P for the isolated footing or the resultant forces ΣP for the combined footing lies as far as possible on the center of gravity of the footing. Then, the footing area A_f is determined from $A_f = \Sigma P/\ (q_{net})_{all}$. It must be considered also that the contact pressure is uniform for all footings and nearly is the same. Table 71 shows the load on the footing P, footing area A_f and net contact pressure f_n between the footing and the soil.

6.2 Dimensions of tie beams

Tie beams are chosen so that their axes coincide with those of columns to avoid the torsion. The width of the tie beam is chosen to be not longer than the smallest column side, d_g = 0.30 [m], while the depth of the tie beam is chosen to be at least double of its width to make it stiff enough, d_g = 0.6 [m]. Tie beams for all footings have a constant rectangular section of 0.3 [m] × 0.6 [m]. It is considered that footings and tie beams are resting on the soil and there is no looseness of the contact pressure between them and the soil.

Table 71 Load ΣP, footing areas A_f and net contact pressures f_n

Footing	Total load on footing ΣP [kN]	Footing area A_f [m^2]	Net contact pressure f_n [kN/ m^2]
F1	1054	5.0 × 1.1	192
F2	730	2.0 × 2.0	183
F3	450	1.5 × 1.5	200
F4	1924	5.0 × 2.0	192
F5	1276	2.6 × 2.6	189
F6	730	2.0 × 2.0	183
F7	364	1.4 × 1.4	186
F8	648	1.8 × 1.8	200
F9	324	2.1 × 0.8	193
F10	881	2.1 × 2.1	200
F11	304	1.3 × 1.3	180

6.3 Thickness of footings

The footing thickness and reinforcement are designed according to the Egyptian code of practice ECP, working stress method. In this case, the reinforcement concrete section must resist the working stress acting on it safely such as the shear stress, punching stress, bond stress and bending moment. It is expected that the stresses of shear, punching and bond for the footings connected with tie beams are strong enough to resist the permissible stresses. Consequently, there is no requirement to check on these stresses and it is sufficient only to check on the bending moment to determine the thickness of the footings, tie beams and reinforcement.

The thickness of the footing in this example is chosen to fulfill the safety conditions at the analysis of the footing whether they are connected with or without tie beams excepting the reinforcement, which is chosen for every structural system separately.

The first step in the design is determination of the primary footing thickness from the depth d_p that resists the punching stress. This depth is mostly the critical depth for the isolated footing.

The depth to resist punching shear d_p [m] is given by

$$d_p = \frac{Q_p}{b_o q_{pall}} \qquad \text{(i)}$$

where:

b_o Perimeter of critical punching shear section around the column considering the position of the column wherever the column at the edge, corner or inside [m]
Q_p Punching force [kN], $Q_p = P_{col} - A_p \,.\, f_n$
A_p Punching area [m^2], for simplicity $A_p = A_{col}$
A_{col} Cross section of the column [m^2]
P_{col} Column load [kN]
f_n Average contact pressure between the footing und the soil under the column [kN/ m^2]
q_{pall} Allowable concrete punching strength [MN/ m^2]

The allowable concrete punching strength q_{pall} [MN/ m^2] is given by

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp}\,, \leq q_{cp} \qquad \text{(ii)}$$

where:

q_{cp} Main value of shear strength [MN/ m^2], q_{cp} = 0.9 [MN/ m^2]
b, a Column sides [m]

The allowable concrete punching strength for the columns those have the greatest cross section (a × b = 0.25 × 0.6 [m^2]) will be q_{pall} = 0.825 [MN/ m^2] while for the other columns will be q_{pall} = 0.9 [MN/ m^2].

Substituting the value of allowable punching shear strength q_{pall} in Equation (i) leads to an equation of second order in the unknown d_p. Solving this equation gives the depth d_p that is required to resist the punching shear as shown in Table 72. This depth, addition to the concrete cover for the nearest 10 [cm], is chosen as a primary data for the footing thickness, considering that the minimum footing thickness is 30 [cm]. After carrying out the analysis, this depth may be modified if necessary to fulfill the condition of safety against the remaining shear, bond, bending moment stresses.

Table 72 Determination of the footing depth d_d to resist the punching shear

Footing	Load P [kN]	Net contact pressure f_n [kN/ m^2]	Column section $A_{col} = a \times b$ [m^2]	Punching load $Q_p = P - f_n \times A_{col}$ [kN]	Punching depth d_p [m]	Chosen depth d_d [m]
F1	730	192	0.25 × 0.40	711	0.31	0.35
F2	730	183	0.25 × 0.40	712	0.31	0.35
F3	450	200	0.25 × 0.25	437	0.25	0.25
F4	1276	192	0.25 × 0.60	1247	0.44	0.45
F5	1276	189	0.25 × 0.60	1248	0.44	0.45
F6	730	183	0.25 × 0.40	712	0.31	0.35
F7	364	186	0.25 × 0.25	352	0.21	0.25
F8	648	200	0.25 × 0.40	628	0.29	0.35
F9	162	193	0.25 × 0.25	150	0.16	0.25
F10	319	200	0.25 × 0.25	307	0.39	0.45
F11	304	180	0.25 × 0.25	293	0.19	0.25

Generation of the FE-Net

In regard to the narrowness of the distance between some axes and design dimensions of the footings, columns and tie beams, a refined net of finite elements is used. It is necessary to consider the following notes when generating the FE-Net.

- Generate a homogenous mesh over the whole foundation area as possible as you could

- Element size is chosen to be equal the foundation thickness if possible

- Switching from a small element to a large one must be done gradually so that the difference between the side of the element and that of its neighboring element is not larger than the double in both directions

- The net of the finite elements is generated for the entire area firstly, and then the unnecessary elements are removed to define the foundation shape. The footings are represented by plate elements while the tie beams are represented by beam elements. It is not allowed to leave a beam element separately without connection with a plate element because the mean element used in the program *ELPLA* is the plate element

- In the program *ELPLA*, loads may be applied to the net of the finite elements outside nodes at any position independently from the element sizes

- As the tie beam is represented by beam elements, the width of the plate element adjacent to the beam element is chosen to be half the width of the tie beam. Consequently, the soil effect on the area around beam nodes will be equivalent to that on the actual contact area of the tie beam

- In spite of the plate elements must join with beam elements in free places among footings, but it is easy to cancel its effect quietly. This can be done by assuming that the modulus of elasticity or the thickness of the plate elements equal to zero, then the program will cancel their effect automatically

- Beam elements may be placed in *x*- or *y*-direction on the net connected to plate elements at their nodes to represent tie beams in *x*- or *y*-direction. Diagonal tie beams are represented by diagonal beam elements. Each diagonal beam element may be placed on the nearest two diagonal nodes

- Using the advantage of generating all footings on one net, it is easy to take a combined section for a group of footings indicating the internal forces, settlements and contact pressures

- It is possible at the analysis of isolated footings without tie beams to generate an independent net for each footing, but it is preferred to generate one net for the whole foundation area considering all footings to save the effort for constructing another net at the analysis of a group of footings connected with tie beams

6.4 Creation of loads on the net

Creation of loads on the net may be carried out by one of the following two ways:

- Considering the column load as a point load on the net to simplify the editing of the input data

  Consequently, the critical positive moment under the column will be calculated at the column face. Due to the small size of the finite element, it is expected that the moment under the point load will be so high

- Converting the concentrated column load P to an equivalent distributed load P_w acting on the centerline of the footing thickness with slope 1:1 such as

$$P_w = \frac{P}{(a+t)(b+t)} \qquad \text{(iii)}$$

where:
a, b Column sides [m]
t Footing thickness [m]

In this case the critical positive moment and area of reinforcement steel must be determined under the column directly. The second way for creation the loads is considered in this example. Figure 88 shows the net of the finite elements for the isolated footings without tie beams while Figure 89 shows that of a group of footings connected with tie beams. Table 73 shows the conversion of the concentrated column load *P* to an equivalent distributed load P_w.

Table 73 Conversion of the concentrated column load *P* to an equivalent distributed load P_w

Column	Load P [kN]	Coordinate x [m]	 y [m]	Column section $A_{col} = a \times b$ [m²]	Footing thickness t [m]	Distributed area $A_w = (a + t) \times (b + t)$ [m²]	Distributed load $P_w = P/ A_w$ [kN/ m²]
A-1	324	0.125	10.05	0.25 × 0.25	0.40	0.45×0.65	1108
C-1	730	3.75	10.05	0.25 × 0.40	0.40	0.65×0.80	1404
E-1	730	8.25	10.05	0.25 × 0.40	0.40	0.65×0.80	1404
G-1	450	11.85	10.05	0.25 × 0.25	0.30	0.55×0.55	1488
A-2	648	0.20	6.45	0.25 × 0.40	0.50	0.65×0.75	1329
C-2	1276	3.75	6.45	0.25 × 0.60	0.50	0.75×1.10	1547
E-2	1276	8.25	6.45	0.25 × 0.60	0.50	0.75×1.10	1547
G-2	730	11.85	6.45	0.25 × 0.40	0.40	0.65×0.80	1404
C-3	364	3.75	3.75	0.25 × 0.25	0.30	0.55×0.55	1203
D-3	648	7.35	3.75	0.25 × 0.40	0.40	0.65×0.80	1246
A-4	162	0.125	2.85	0.25 × 0.25	0.30	0.40×0.55	736
B-4	162	1.95	2.85	0.25 × 0.25	0.30	0.40×0.55	736
D-5	319	7.35	1.95	0.25 × 0.25	0.50	0.50×0.50	1276
F-5	340	9.15	1.95	0.25 × 0.25	0.50	0.50×0.50	960
G-5	304	11.85	1.95	0.25 × 0.25	0.30	0.55×0.55	1005
D-6	161	7.35	0.125	0.25 × 0.25	0.50	0.50×0.50	644
F-6	161	9.15	0.125	0.25 × 0.25	0.50	0.50×0.50	644

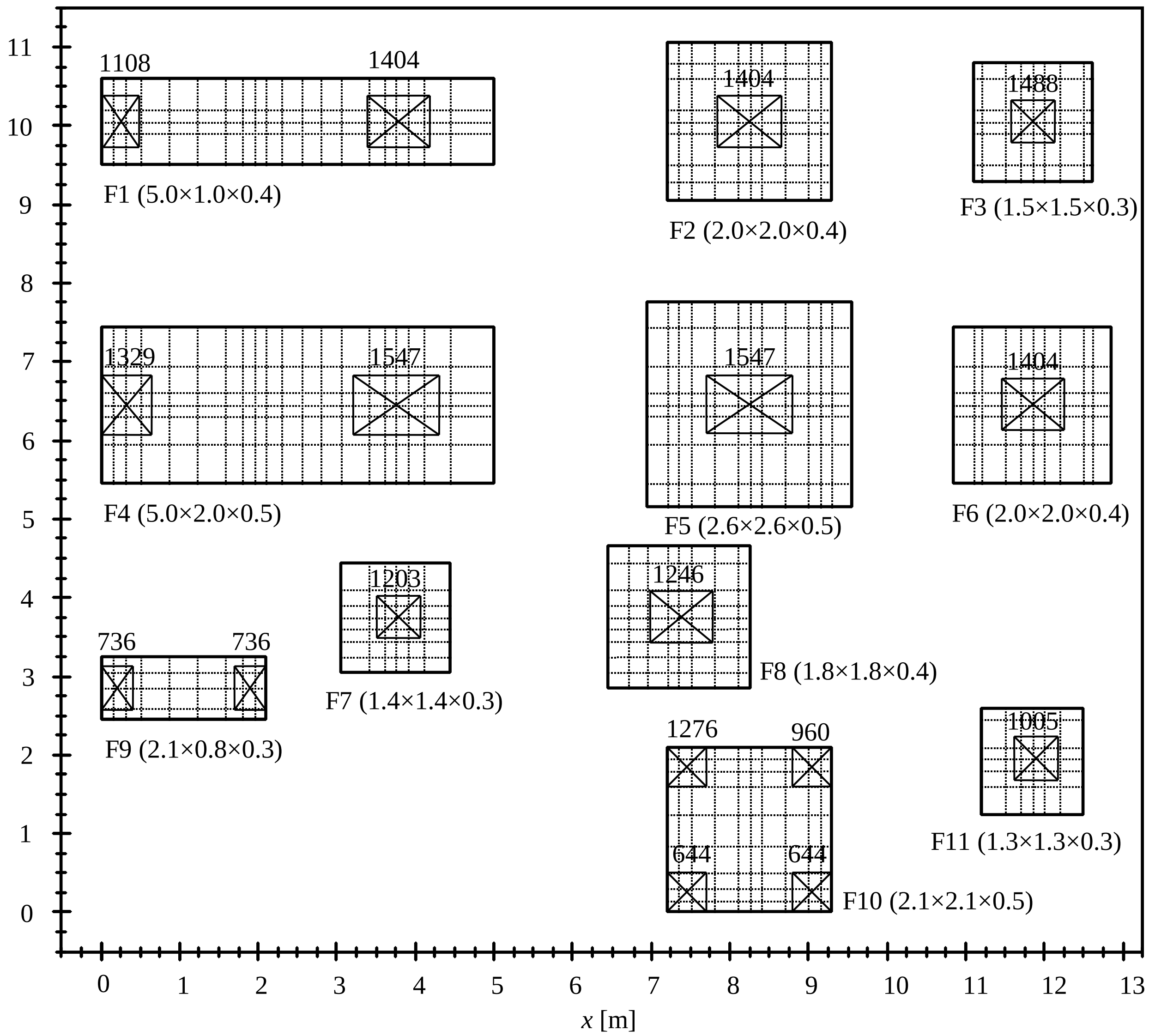


Figure 88 FE-Net of footings without tie beams, loads [kN/ m^2] and footing dimensions [m]

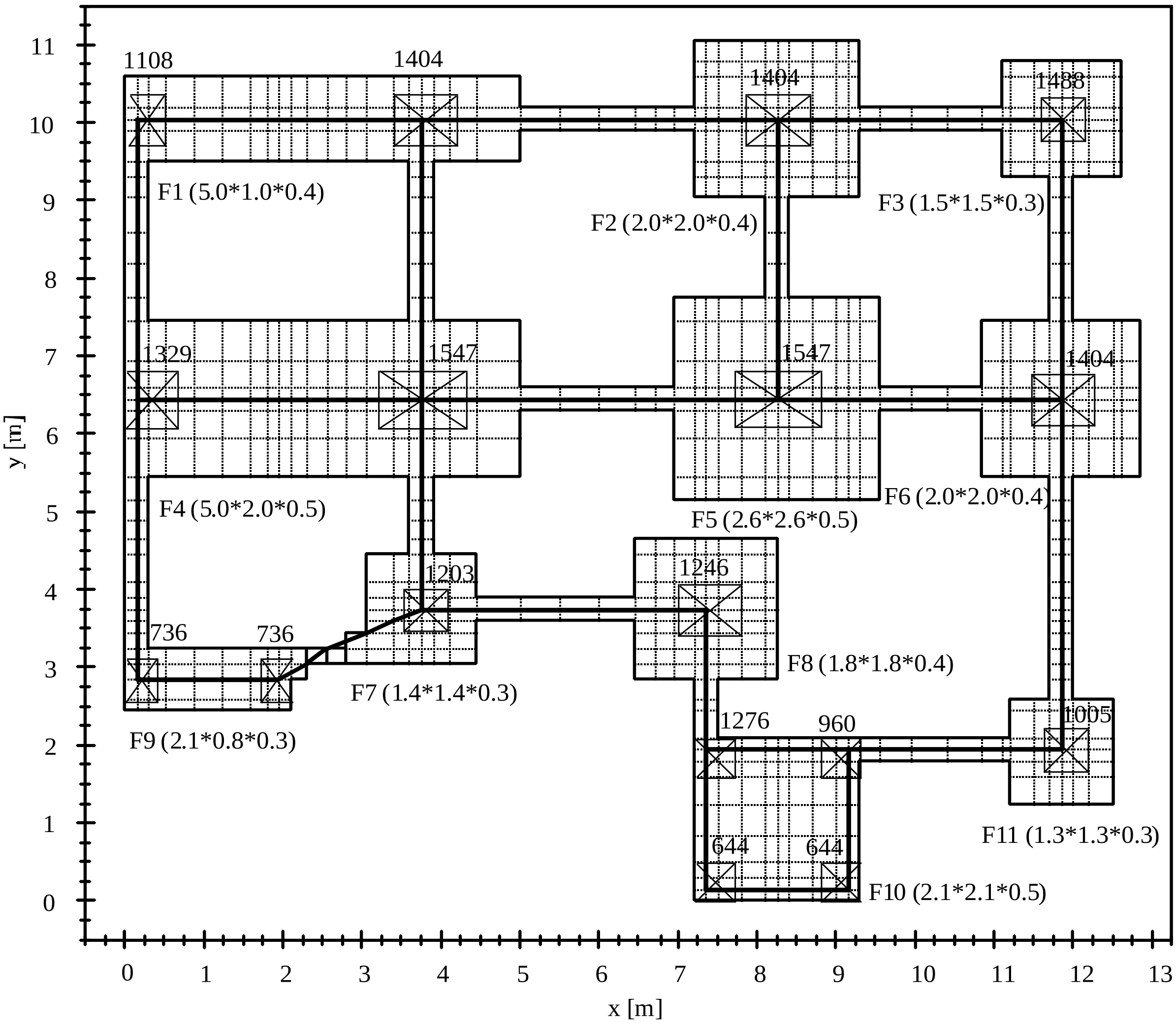


Figure 89 FE-Net of footings with tie beams, loads [kN/ m^2] and footing dimensions [m]

6.5 Reinforcement steel for isolated footings

The area of reinforcement steel A_s is given by

$$A_s = \frac{M}{k_2 d_m} \qquad \text{(iv)}$$

It is required firstly to check if the footing depth to resist punching shear is also sufficient to resist the bending moment at the critical section according to Equation (v)

$$d_m = k_1 \sqrt{\frac{M}{b}} \qquad \text{(v)}$$

where:

M	Moment at a section obtained from analysis [MN.m]
b	Width of the section to be designed [m], b = 1.0 [m]
d_m	Depth required to resist the moment [m]
k_1 and k_2	Coefficients according to ECP

The program *ELPLA* gives the results of the bending moments per meter in both directions x and y and also the values of areas of reinforcement steel at all nodes of the net of finite elements. Figure 90 shows bending moments m_x while Figure 91 shows bending moments m_y for the critical sections in directions x and y, respectively. Table 74 and Table 75 show check depth required to resist the bending moment and also the area of reinforcement steel that is required for the critical section in case of analysis of isolated footings.

Table 74 Check depth required to resist the bending moment and determination of the area of reinforcement steel in *x*-direction

Footing	Moment m_x [kN.m/ m]		f_c [kg/ cm^2]	Required area of steel A_s [cm^2/ m]		Chosen steel [Rft/ m]	
	- ve m_x	+ ve m_x		A_{sx1} Top	A_{sx2} Bottom	A_{sx1} Top	A_{sx2} Bottom
F1	175	124	85	28.77	19.94	10 Φ 19	10 Φ 16
F2	-	82	55	-	12.98	-	7 Φ 16
F3	-	50	60	-	11.15	-	6 Φ 16
F4	181	112	65	22.55	13.64	8 Φ 19	8 Φ 16
F5	-	126	50	-	15.38	-	8 Φ 16
F6	-	76	50	-	11.93	-	6 Φ 16
F7	-	39	60	-	6.68	-	*min* A_s
F8	-	62	45	-	9.64	-	*min* A_s
F9	61	2	70	13.76	-	7 Φ 16	*min* A_s
F10	76	11	50	9.12	1.25	*min* A_s	*min* A_s
F11	-	29	50	-	6.37	-	*min* A_s

The following notes must be considered when reinforcing the footings:

- Suitable reinforcement is to be placed at the places of maximum moments wherever in *x*- or *y*-direction. The reinforcement is chosen to be enough to resist the bending moment. It is not required to determine additional reinforcement to resist the punching shear where it is supposed that the concrete section can resist the punching stress without reinforcement

- The top and bottom reinforcement in both *x*- and *y*-directions at the sections of minimum moments are empirically taken as 0.15 [%] of the concrete cross section. The considered minimum area of reinforcement steel for all footings is

  $$min\ A_s = 6\ \Phi\ 16 = 10.1\ [cm^2/\ m]$$

- For a combined footing for two columns, the calculated reinforcement under the column in the transversal direction is distributed under the column to a distance *d* from the face of the column

- For an isolated footing for a column, it is enough to consider only the bottom reinforcement in both *x*- and *y*-directions

Table 75 Check depth required to resist the bending moment and determination of the area of reinforcement steel in *y*-direction

Footing	Moment m_y [kN.m/ m]		f_c [kg/ cm^2]	Required area of steel A_s [cm^2/ m]		Chosen steel [Rft/ m]	
	- ve m_y	+ ve m_y		A_{sy1} Top	A_{sy2} Bottom	A_{sy1} Top	A_{yx2} Bottom
F1	-	42	45	-	6.53	*min* A_s	*min* A_s
F2	-	88	55	-	13.97	-	7 Φ 16
F3	-	50	55	-	11.08	-	6 Φ 16
F4	-	117	50	-	14.30	*min* A_s	8 Φ 16
F5	-	153	55	-	18.83	-	10 Φ 16
F6	-	85	60	-	13.50	-	7 Φ 16
F7	-	38	50	-	8.36	-	*min* A_s
F8	-	72	50	-	11.30	-	6 Φ 16
F9	-	13	35	-	2.79	*min* A_s	*min* A_s
F10	65	10	30	7.70	1.14	*min* A_s	*min* A_s
F11	-	31	45	-	6.75	-	*min* A_s

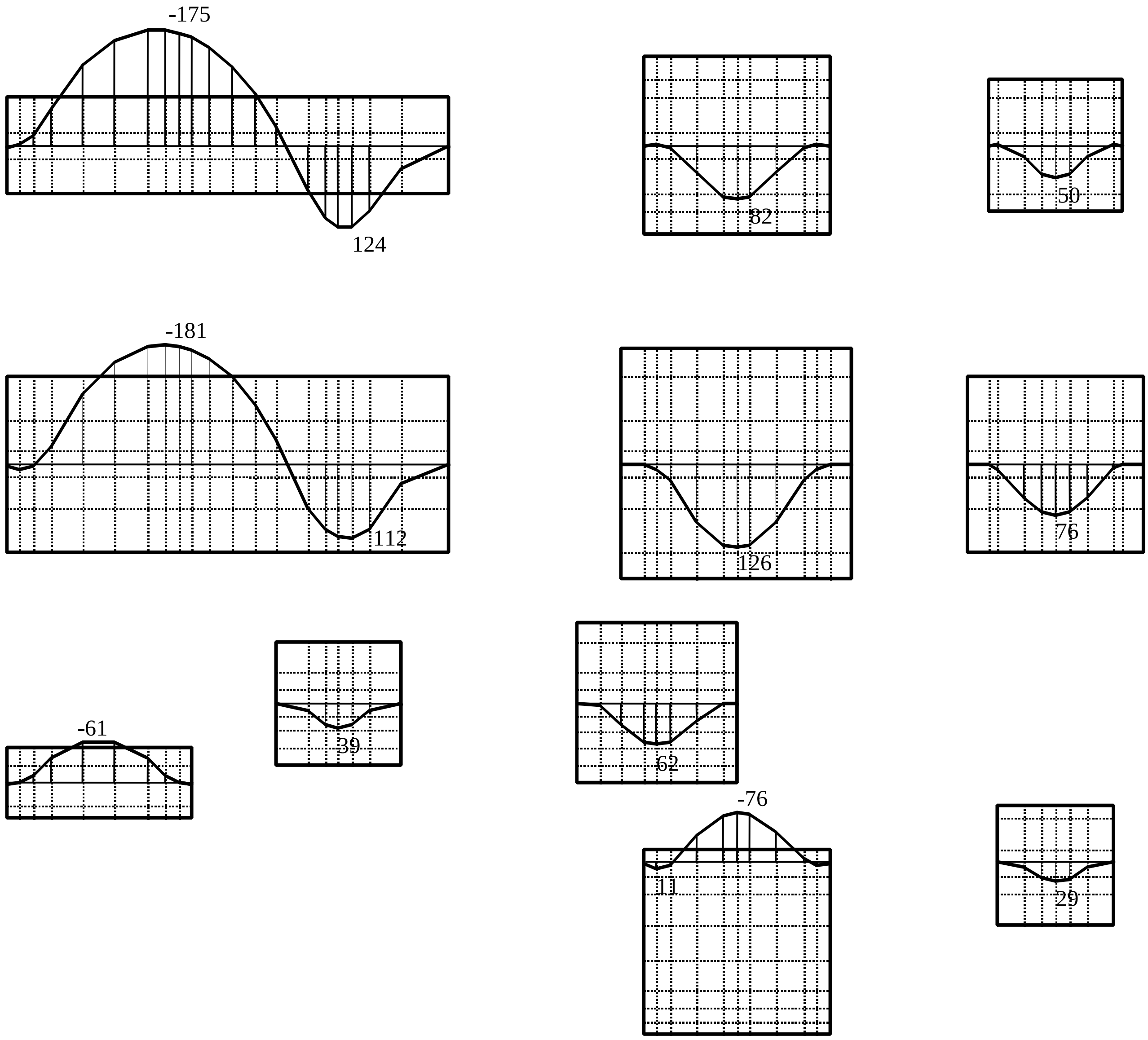


Figure 90 Moment m_x [kN.m/ m] at critical sections on the footings

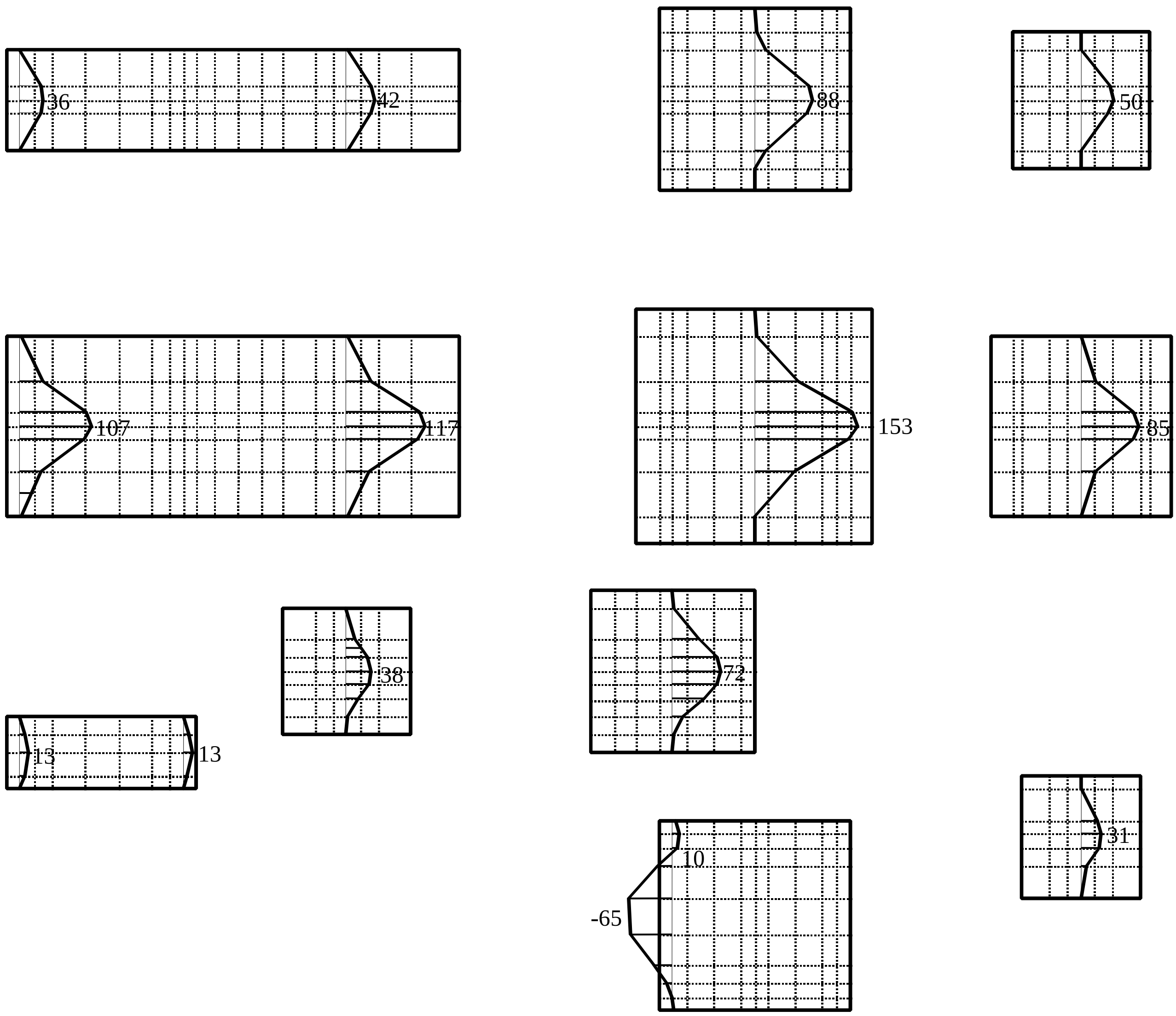


Figure 91 Moment m_y [kN.m/ m] at critical sections on the footings

6.6 Check shear stress for isolated footings

It is required for isolated footings to check if the shear stress q_{sh} in the footing does not exceed the allowable shear stress of concrete q_c = 0.9 [MN/ m^2]. The shear stress q_{sh} [MN/ m^2] is given by

$$q_{sh} = \frac{Q_{sh}}{b\, d_{sh}} \qquad \text{(vi)}$$

where:

Q_{sh} Shearing force at critical section of shear. The program *ELPLA* gives Q_{sh} per meter at all nodes of the net in both *x*- and *y*-directions [MN/ m]

d_{sh} Depth required to resist shear stress [m]

b Width of critical section of shear, b = 1.0 [m], where Q_{sh} is per meter

Figure 92 shows the shearing force Q_{sh} in *x*-direction while Figure 93 shows that in *y*-direction at the critical sections. Table 76 shows check depth required to resist shear stress. The depths for all footings are save in shear stress.

Table 76 Check depth required to resist shear stress

Footing	Footing depth	*x*-direction		*y*-direction	
	d_{sh} [m]	Q_x [MN/ m]	$q_{sh} = \frac{Q_x}{b\, d_{sh}}$ [MN/ m^2]	Q_y [MN/ m]	$q_{sh} = \frac{Q_y}{b\, d_{sh}}$ [MN/ m^2]
F1	0.35	0.252	0.72	0.980	0.28
F2	0.35	0.157	0.45	0.148	0.42
F3	0.25	0.120	0.48	0.910	0.36
F4	0.45	0.308	0.68	0.170	0.38
F5	0.45	0.214	0.48	0.211	0.47
F6	0.35	0.158	0.45	0.129	0.37
F7	0.25	0.900	0.36	0.109	0.44
F8	0.35	0.137	0.39	0.160	0.46
F9	0.25	0.100	0.40	0.370	0.15
F10	0.45	0.133	0.30	0.135	0.30
F11	0.25	0.700	0.24	0.720	0.29

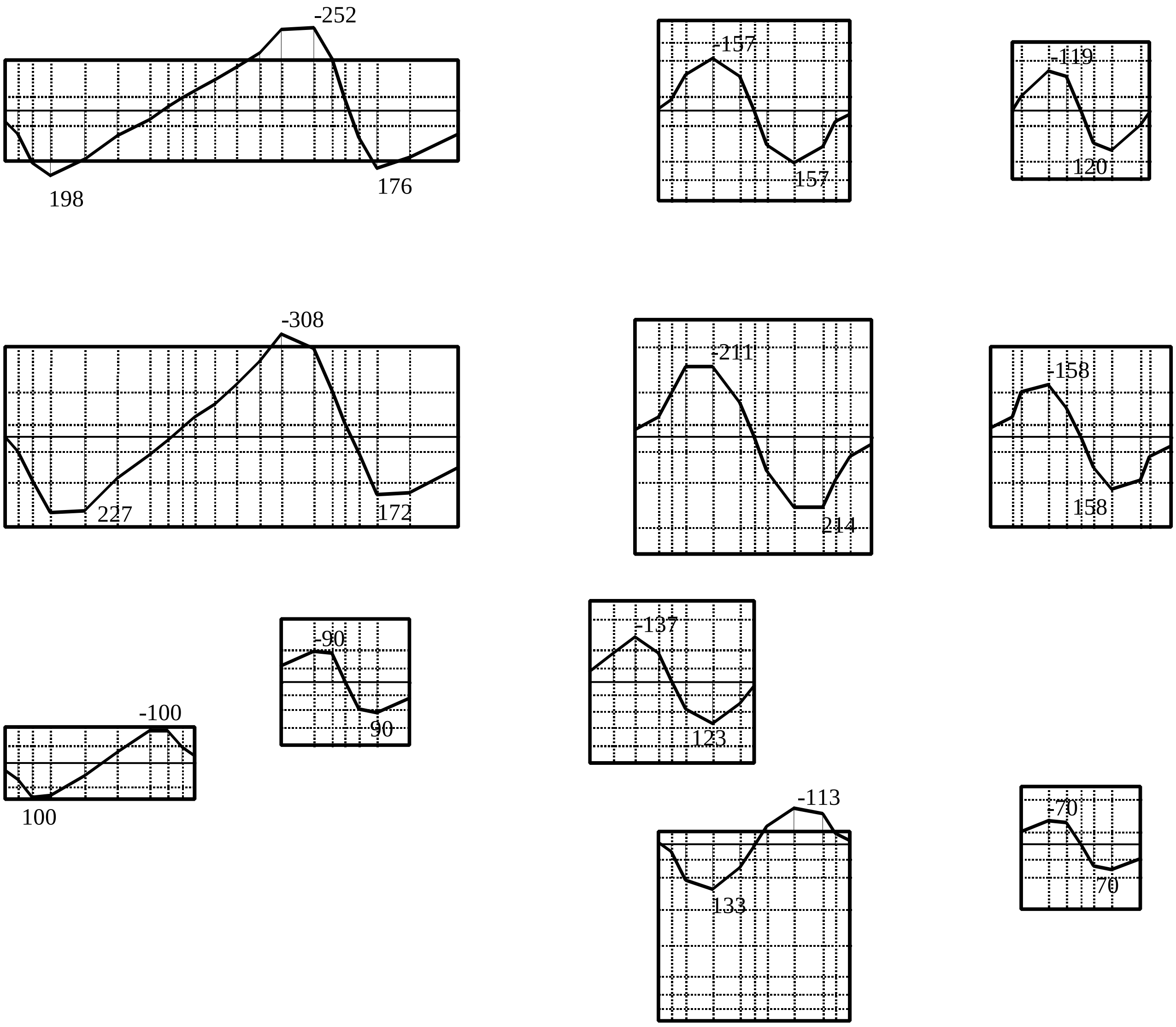


Figure 92 Shearing force Q_x [kN/ m] at critical sections on the footings

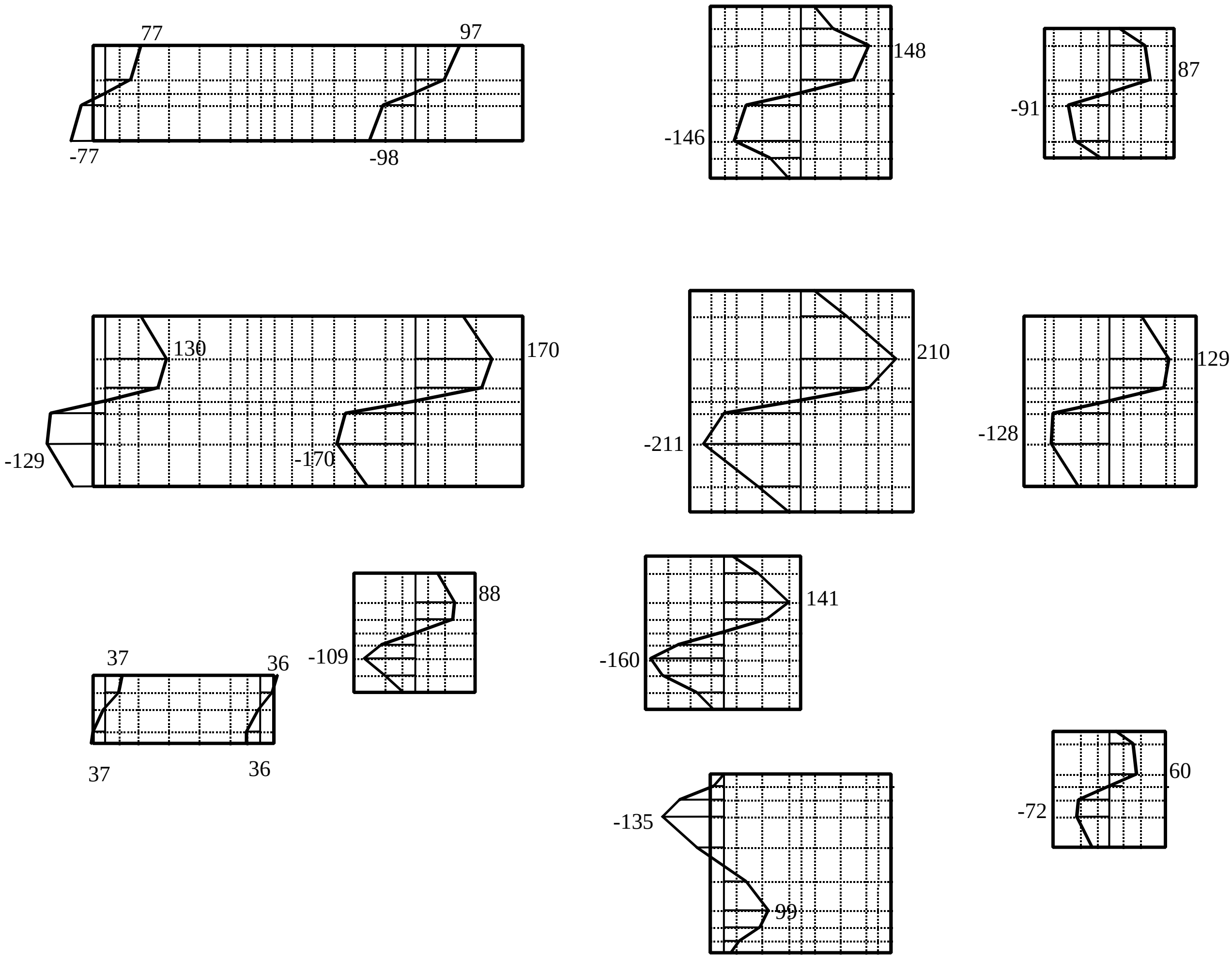


Figure 93 Shearing force Q_y [kN/ m] at critical sections on the footings

6.7 Check bond stress for isolated footings

It is required also to check if the bond stress between the reinforcement steel and the concrete does not exceed the allowable bond stress q_b = 1.2 [MN/ m^2]. The bond stress q_{bo} [MN/ m^2] is given by

$$q_{bo} = \frac{Q_p}{0.87d \sum O} \qquad \text{(vii)}$$

where:

Q_b Shearing force at section of maximum bending moment [MN]
Shearing force for an isolated footing of a column is $Q_b = 0.25\ (P_{col} - f_n\ .\ A_{col})$. It is not necessary to check bond stress for the combined footing of two columns or more, because the critical bending moment in this case lies at the point of zero shear. Here, the zero shearing force is also the bond force. For simplicity, the bond forces Q_b in both x- and y-directions for all footings are considered equal where the difference in Q_b in both directions is small

d_b Depth at that section [m]

ΣO Sum of the perimeter of main reinforcement steel [m]

The allowable bond stress in this example is for steel bars take L-shape at their ends. Table 77 shows the bond forces for the isolated footings for a column and also check bond stress. Bond stress for all footings lies within the permissible values.

Table 77 Check bond stress for the isolated footings

Footing	Bond force Q_b [MN]	Rft A_{sy2} [/ L]	Sum of perimeter of main Rft ΣO [m]	Footing depth d_b [m]	Bond stress q_{bo} [MN/ m^2]
F2	0.178	14 Φ 16	0.704	0.35	0.83
F3	0.109	9 Φ 16	0.452	0.25	1.11
F5	0.312	21 Φ 16	0.106	0.45	0.75
F6	0.178	12 Φ 16	0.603	0.35	0.97
F7	0.880	9 Φ 13	0.368	0.25	1.10
F8	0.157	9 Φ 16	0.452	0.35	1.14
F11	0.730	8 Φ 13	0.327	0.25	1.03

Figure 94 shows a plan for the isolated footings indicating the footing dimensions and reinforcement with a section at the axis 2-2 after carrying out all processes of the analysis and design for the isolated footings.

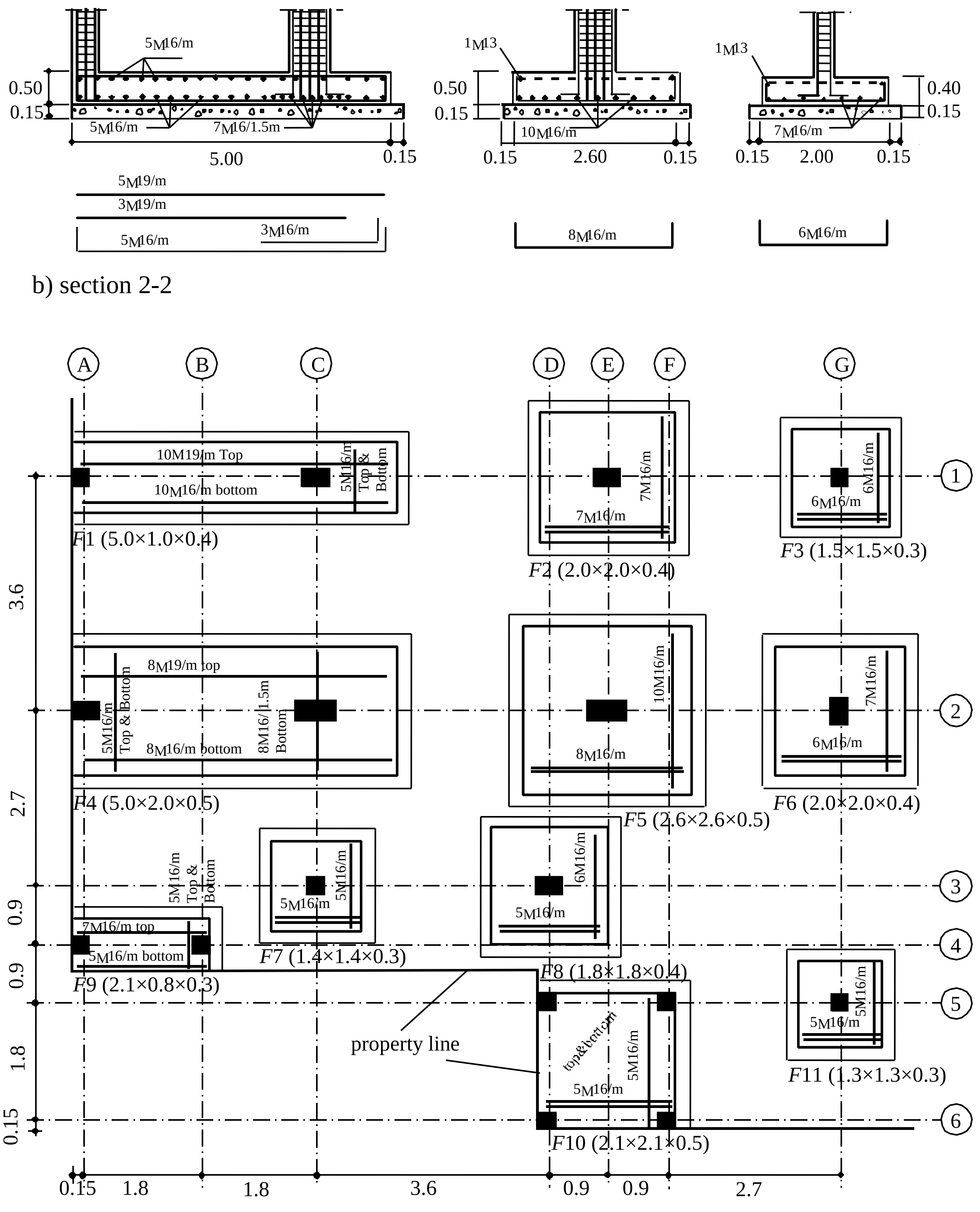


Figure 94 Footing dimensions [m] and reinforcement

6.8 Reinforcement steel for footings connected with tie beams

As mentioned before the thickness of the footing in this example is chosen to fulfill the safety conditions at the analysis of the footing whether they are connected with or without tie beams excepting the reinforcement, which is chosen for every structural system separately. Therefore, the analysis is carried out again for the footings with the same data of the previous footings but with considering the tie beams.

Figure 95 shows the bending moment m_x while Figure 96 shows the bending moment m_y for footings connected with tie beams at the critical sections in *x*- and *y*-directions, respectively. Table 78 and Table 79 show check depth required to resist the bending moment and also the required reinforcement for the critical sections in case of the structural design of the footings connected with tie beams.

Table 78 Check depth required to resist bending moment and determination of reinforcement steel in *x*-direction

Footing	Moment m_x [kN.m/ m]		f_c [kg/ cm^2]	Required area of steel A_s [cm^2/ m]		Chosen steel [Rft/ m]	
	- ve m_x	+ ve m_x		A_{sx1} Top	A_{sx2} Bottom	A_{sx1} Top	A_{sx2} Bottom
F1	79	79	50	12.51	12.45	7 Φ 16	7 Φ 16
F2	-	45	35	-	6.94	-	*min* A_s
F3	-	42	55	-	9.36	-	*min* A_s
F4	116	105	50	14.12	12.70	*min* A_s	*min* A_s
F5	-	97	45	-	11.70	-	*min* A_s
F6	-	59	45	-	9.17	-	*min* A_s
F7	-	16	30	-	3.51	-	*min* A_s
F8	-	75	50	-	11.86	-	*min* A_s
F9	12	2	25	2.51	-	*min* A_s	*min* A_s
F10	25	19	20	2.93	2.19	*min* A_s	*min* A_s
F11	-	20	35	-	4.40	-	-

Table 79 Check depth required to resist bending moment and determination of reinforcement steel in *y*-direction

Footing	Moment m_y [kN.m/ m]		f_c [kg/ cm^2]	Required area of steel A_s [cm^2/ m]		Chosen steel [Rft/ m]	
	- ve m_y	+ ve m_y		A_{sy1} Top	A_{sy2} Bottom	A_{sy1} Top	A_{sy2} Bottom
F1	-	24	25	-	3.56	*min* A_s	*min* A_s
F2	-	68	45	-	10.68	-	6 Φ 16
F3	-	43	55	-	9.58	-	*min* A_s
F4	-	82	40	-	9.82	*min* A_s	*min* A_s
F5	-	141	55	-	17.38	-	9 Φ 16
F6	-	52	40	-	8.05	-	*min* A_s
F7	-	23	35	-	5.04	-	*min* A_s
F8	-	78	50	-	12.21	-	7 Φ 16
F9	-	6	20	-	1.28	*min* A_s	*min* A_s
F10	17	58	30	2.01	6.89	*min* A_s	*min* A_s
F11	-	29	40	-	6.34	-	*min* A_s

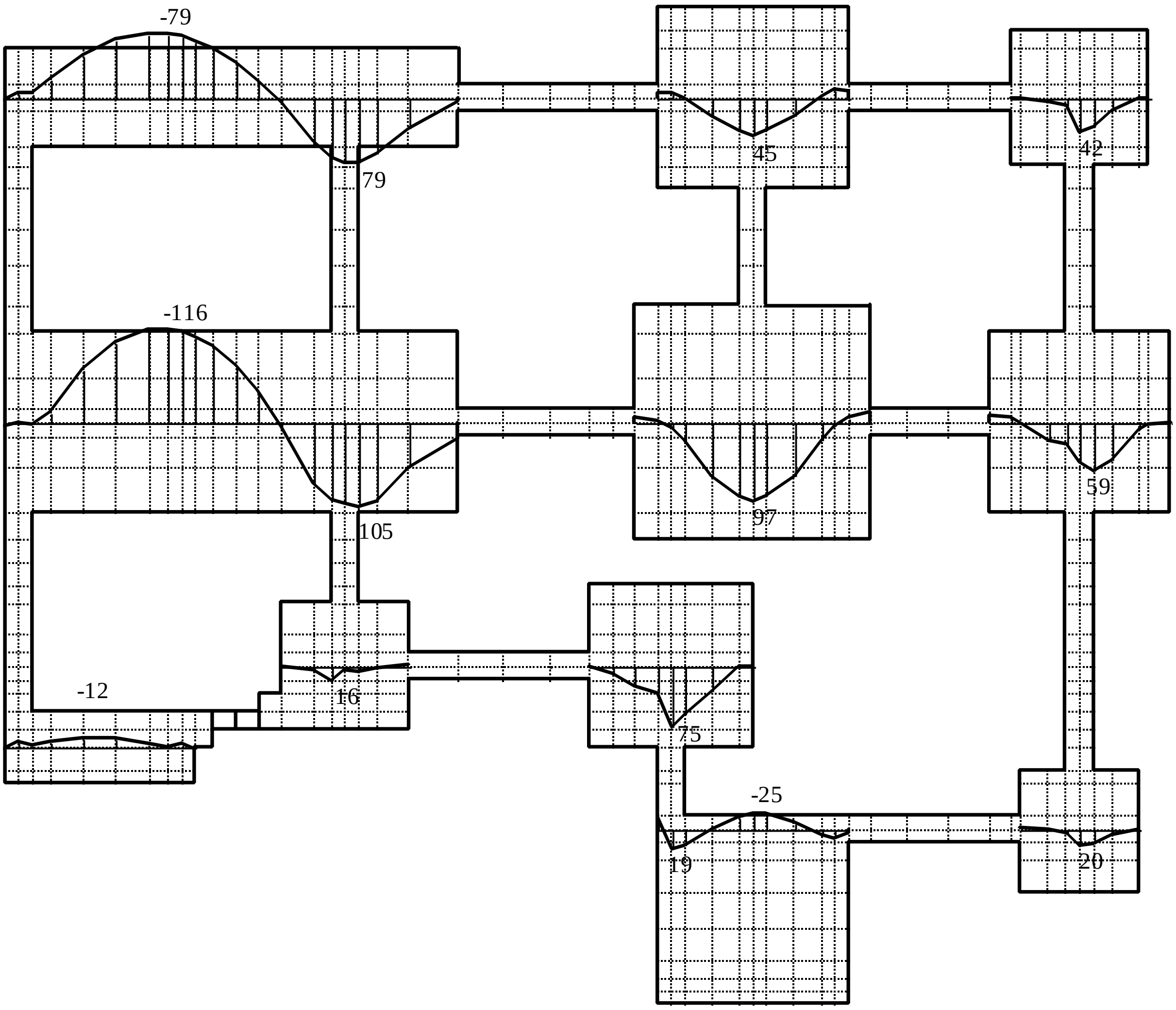


Figure 95 Moment m_x [kN.m/ m] at critical sections on the footings

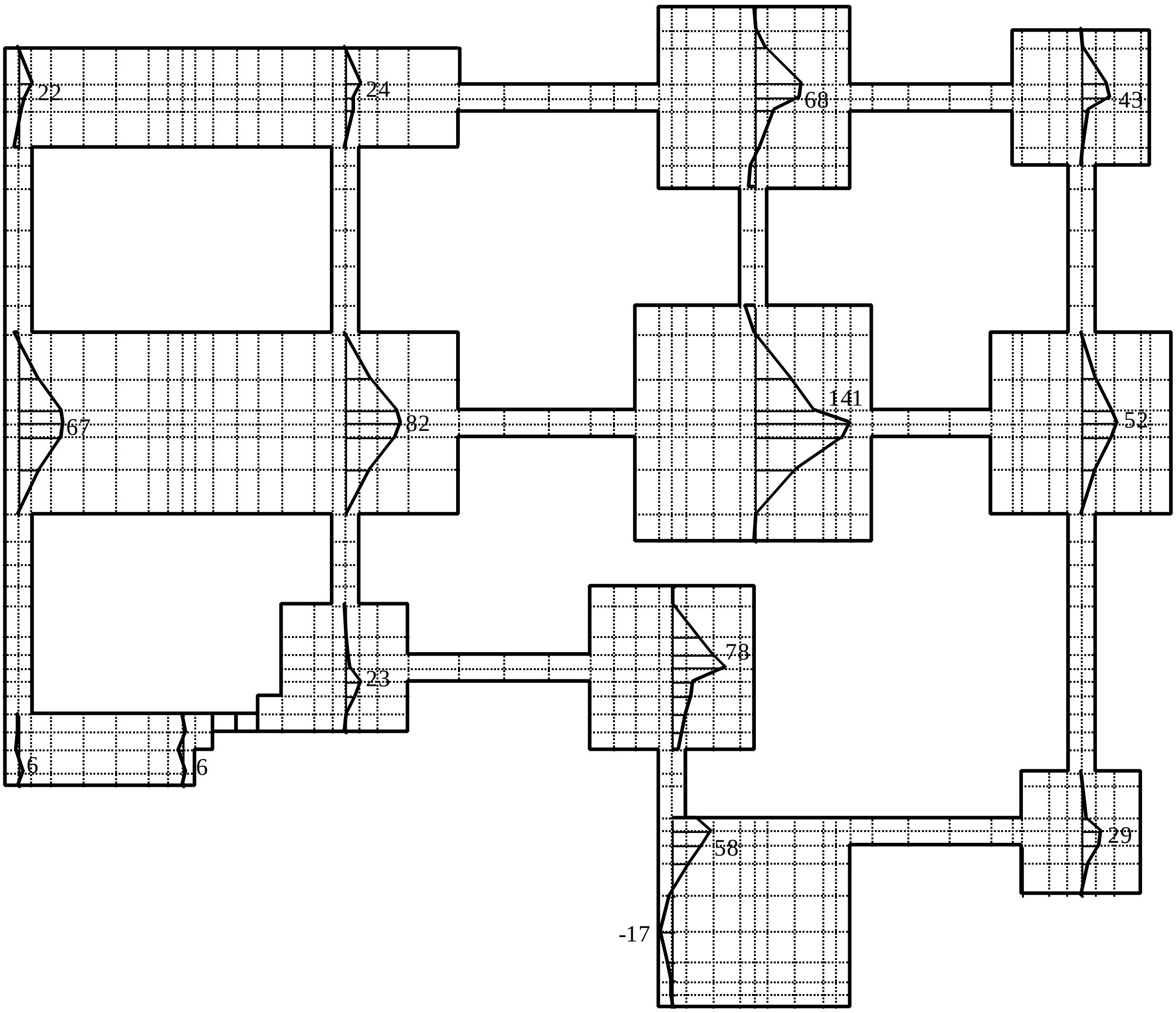


Figure 96 Moment m_y [kN.m/ m] at critical sections on the footings

6.9 Reinforcement steel for tie beams

The cross section of the tie beams is a constant rectangular with the dimension of 0.3 [m] × 0.6 [m]. To simplify the analysis of the tie beams, it will be considered that this section is rectangular also inside the footings and constant at either the compression or tension places. The properties of reinforced concrete and the reinforcement for the tie beams and footings are the same as mentioned before.

The minimum area of top or bottom reinforcement steel in the tie beam is taken as 0.15 [%] of the concrete cross section of the tie beam such as

$min\ A_s = 0.0015\ A_c = 2.7$ [cm^2] Ψ chosen 2 Φ 16/ m = 4.02 [cm^2]

This area of reinforcement steel is sufficient to resist a bending moment of 40 [kN.m]. This minimum area of reinforcement steel will be generally considered for all cross sections of the tie beams besides another additional steel if required at the sections that have bending moments greater than 40 [kN.m].

Figure 97 shows the bending moments M_b for the tie beams in *x*-direction, while Figure 98 shows those in *y*-direction. Table 80 shows the values of bending moments that are greater than 40 [kN.m] and the corresponding additional steel to resist them. Besides, the amount of the additional steel that is required to resist each moment with the definition of its place.

Table 80 Additional reinforcement steel for the tie beams

Moment M_b [kN.m]	Required area of steel [cm^2]	Chosen additional steel [Rft]	Footing	Direction
82	8.35	3 Φ 16	F1	Top/ longitudinal
79	8.13	3 Φ 16	F1	Bottom/ longitudinal
62	6.23	2 Φ 16	F4	Top/ longitudinal
49	4.97	1 Φ 16	F4	Bottom/ longitudinal
45	4.60	1 Φ 16	F6	Bottom/ transversal
51	5.18	1 Φ 16	F10	Bottom/ transversal

Figure 99 shows a group of footings connected with tie beams after completion of its design with a plan for reinforcement and a cross section in the tie beams.

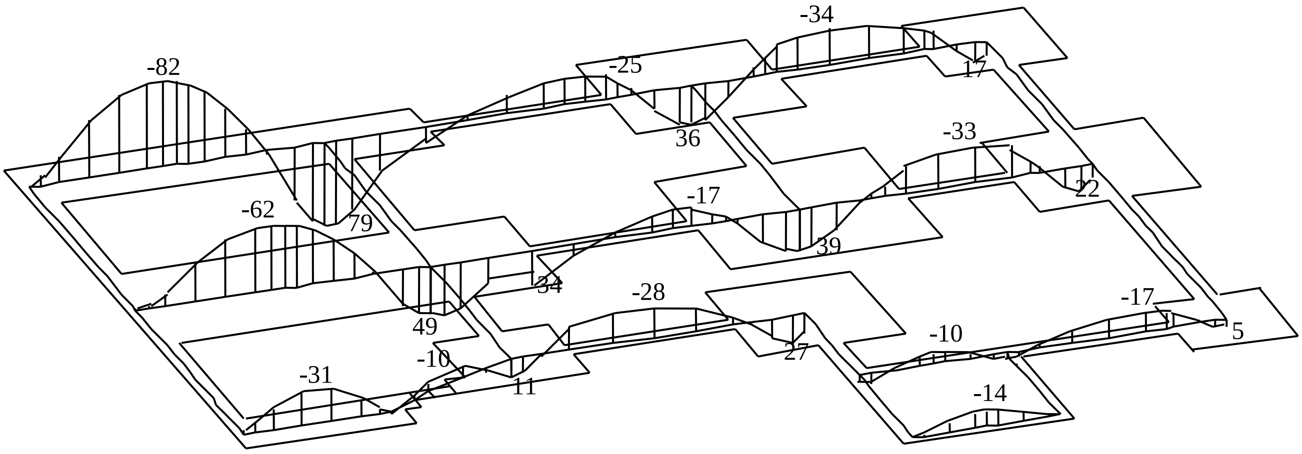


Figure 97 Moment M_b [kN.m] in girders at longitudinal and diagonal directions

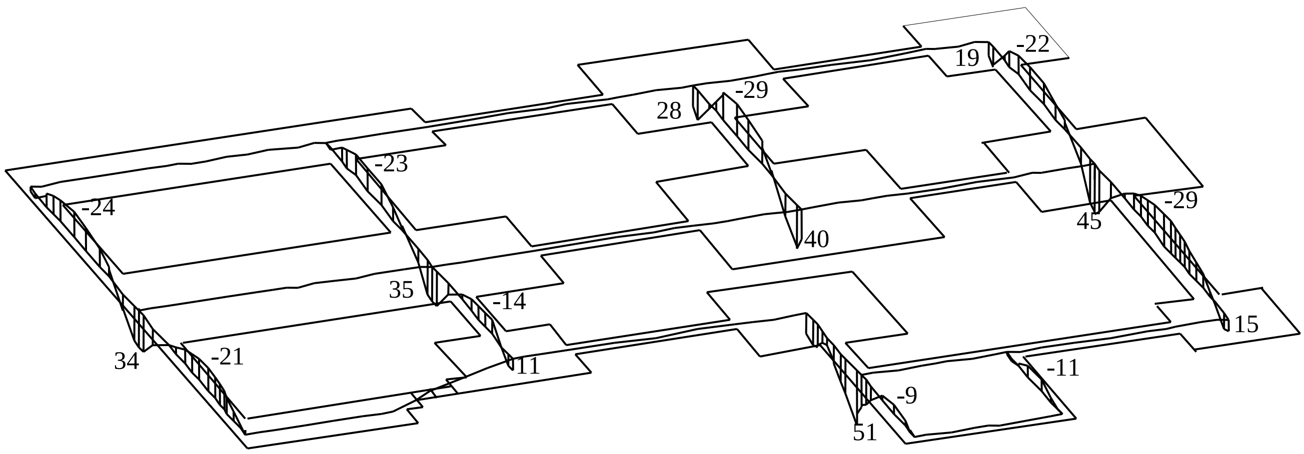
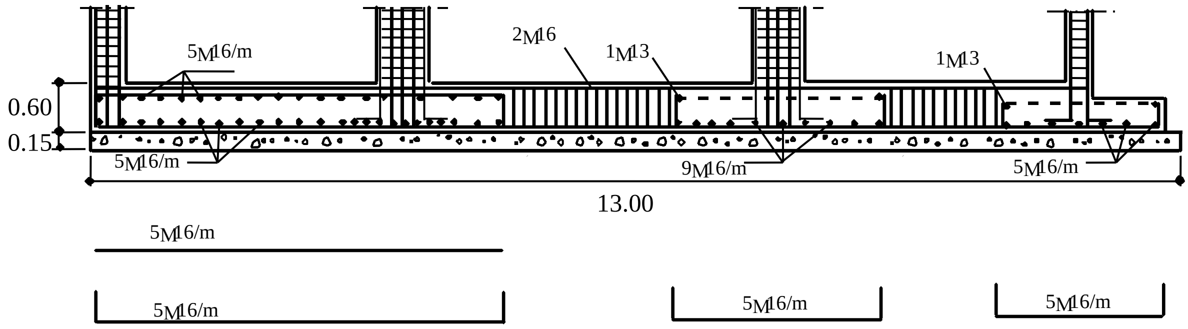
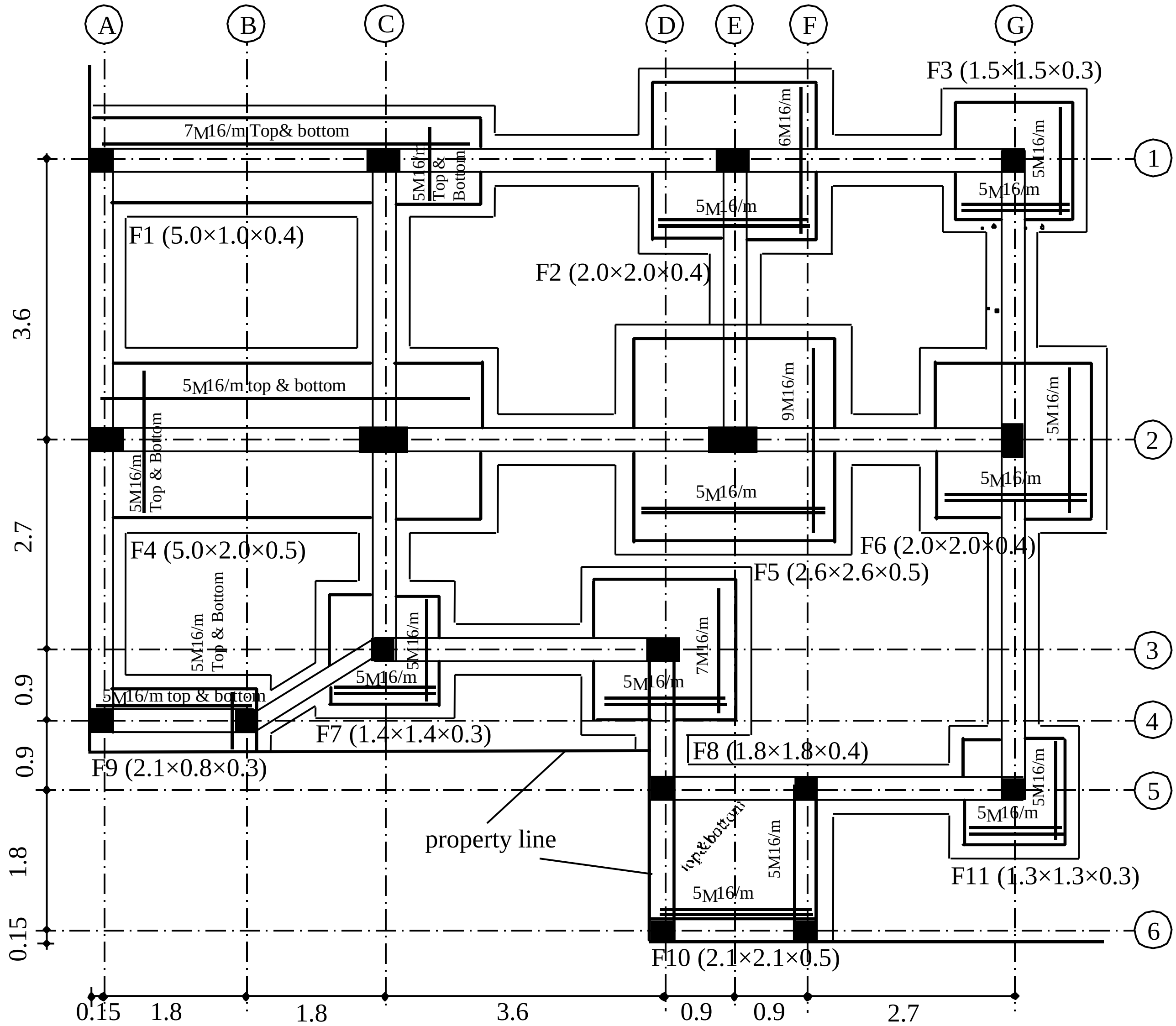


Figure 98 Moment M_b [kN.m] in girders at transversal direction



b) section 2-2



a) plan

Figure 99 Footing dimensions [m], tie beams and reinforcement

7 Comparison between the two structural systems of the footings

7.1 Settlement

Figure 100 shows the settlement at the axis 2-2 for either isolated footings or footings connected with tie beams. It can be observed from this figure that the settlement curve was sharp before connecting the footings, then it becomes much uniform after connecting the footings. Furthermore, the values of settlements decrease greatly, where the maximum settlement at this axis decreases from 0.57 [cm] to 0.47 [cm] with percentage 21 [%].

7.2 Contact pressure

Due to the presence of tie beams also resting on the soil, the contact area between the foundation and the soil increases from 45.75 [m^2] to 53.59 [m^2] with percentage 17 [%]. This contact area will perform certainly to reduce the contact pressure between the soil and the foundation. Consequently, the main contact pressure becomes q_o = 164 [kN/ m^2] instead of 192 [kN/ m^2].

From the assumption of *Winkler's* model that the contact pressure between the soil and the foundation is proportionally at any node with the settlement at that node ($q = k_s.s$), therefore Figure 100, which represents the settlement at axis 2-2, represents also the contact pressure between the soil and foundation at that axis if the value of settlement is multiplied by the modulus of subgrade reaction k_s. It is clear from this figure that the contact pressure between the foundation and the soil, which represents soil reaction, became more uniform due to the presence of tie beams. The maximum contact pressure at this axis decreases with percentage 21 [%] as in case of the settlement.

7.3 Bending moment

The amount of reinforcement steel in the footings is determined according to the bending moment. It can be found from the comparison between the design of footings with and without tie beams that the amount of reinforcement steel decreases to minimum reinforcement due to the presence of the tie beams at the most sections. This is clear in Figure 101, which represents the bending moment at the axis 2-2 where the maximum bending moment m_x decreases from 181 [kN.m/ m] to 116 [kN.m/ m] with a great percentage 56 [%].

7.4 Shearing force

There is no need to check shear stress for footings connected with tie beams where the presence of the tie beams and their reinforcement steel inside the footings resist greatly the shear stress. It is observed that the shearing force decreases greatly as it is indicated in Figure 102, which shows the shearing force at the axis 2-2. Due to the presence of the tie beams the maximum shearing force Q_x decreases from 308 [kN/ m] to 199 [kN/ m] with percentage 55 [%].

8 Conclusion

From the previous analyses, it can be concluded that the design of a group of footings connected with stiff tie beams improves greatly the behavior of these footings toward deformation and rotation. Besides, it decreases the amount of reinforcement steel at the most sections.

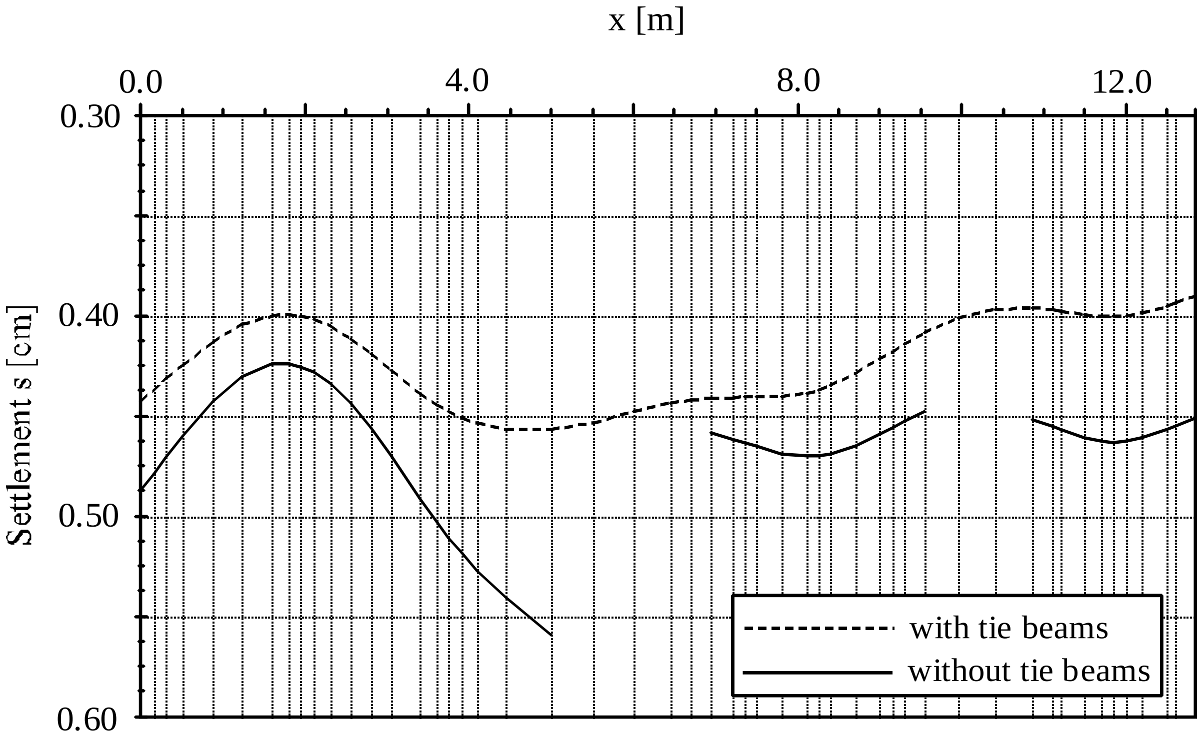


Figure 100 Settlement *s* [cm] at section 2-2

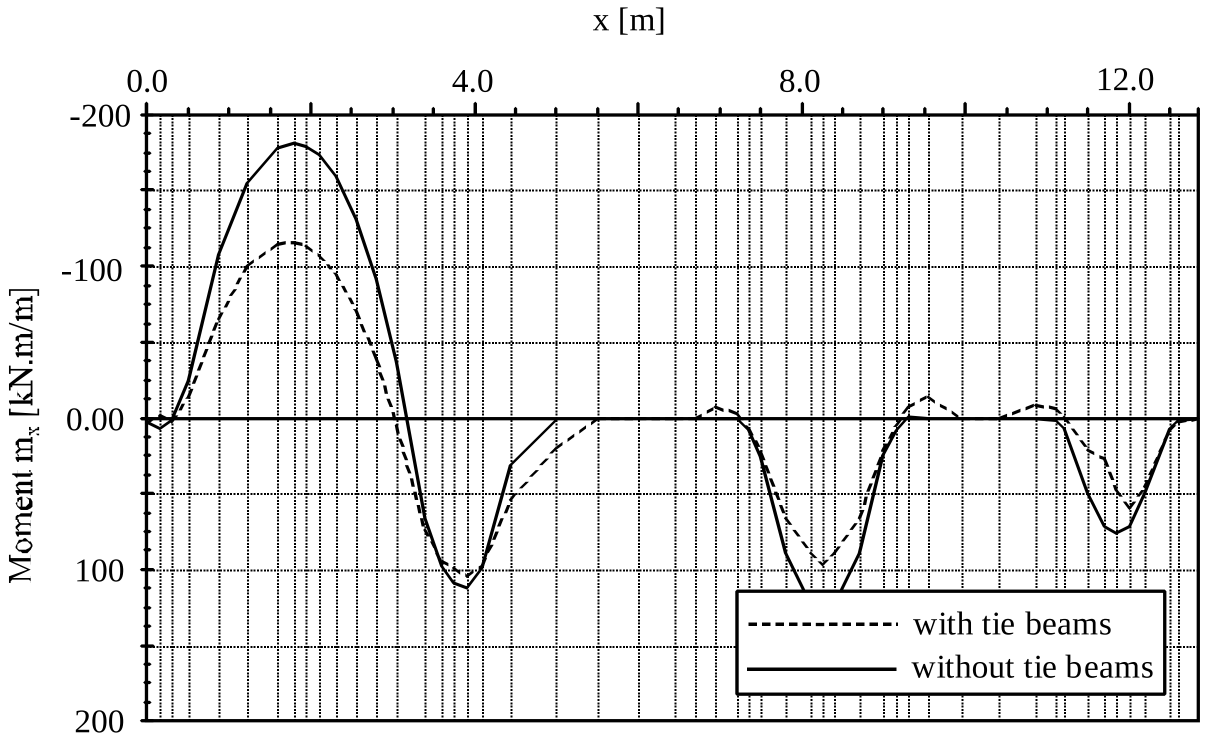


Figure 101 Moment m_x [kN.m/ m] at section 2-2

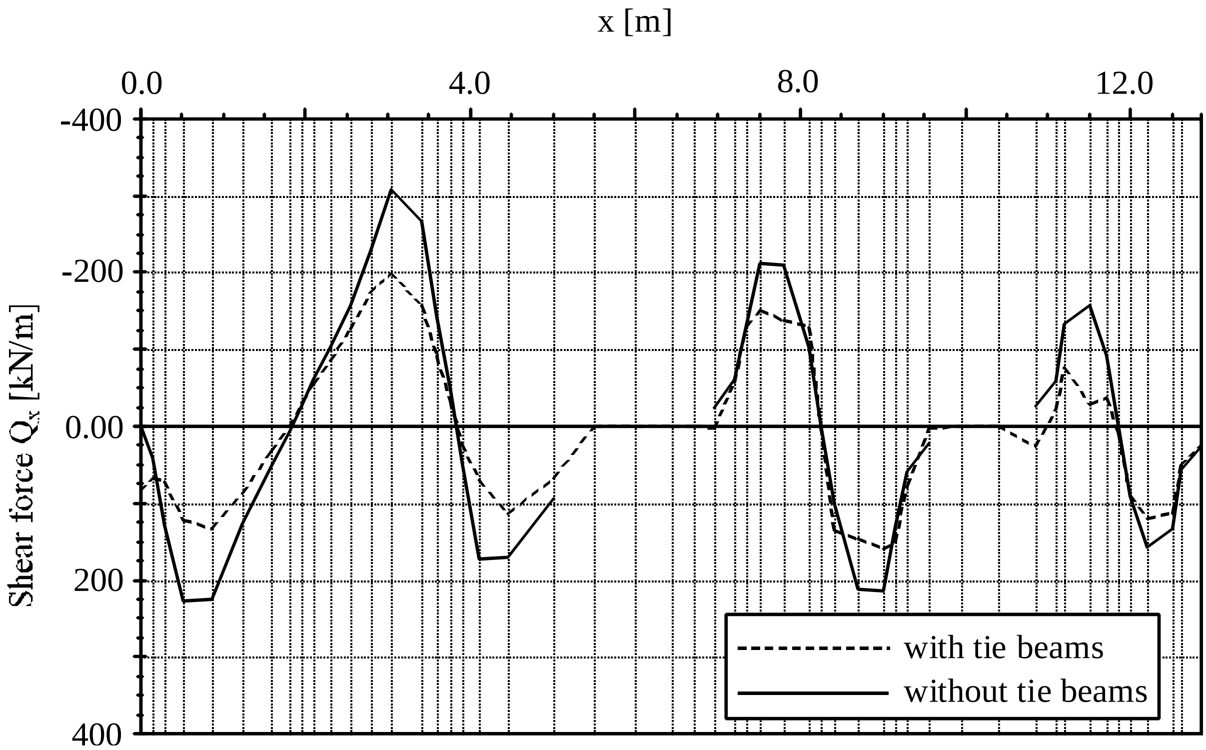


Figure 102 Shearing force Q_x [kN/ m] at section 2-2